Special-Relativistic Resolution of Ehrenfest's Paradox: Comments on Some Recent Statements by T. E. Phipps, Jr.

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It is shown how a consistent kinematic resolution of Ehrenfest's paradox may be given in accordance with the special theory of relativity. Some statements by T. E. Phipps, Jr., connected with these matters, are commented upon. Problems connected with the relation between stress and strain are solved by a manifestly covariant formulation of Hooke's law.

1. INTRODUCTION

T. E. Phipps, Jr. has recently commented on Ehrenfest's paradox. His statements are interesting, although, in my opinion, somewhat misleading. The importance of the topic treated—is the special theory of relativity able to describe accelerated motion of extended bodies in a logically consistent way?—makes it necessary to give the matters a renewed discussion.

In this paper I will comment on some statements by Phipps, trying to clarify how the questions he raises are treated according to special relativity. A consistent special-relativistic resolution of Ehrenfest's paradox will be formulated including a manifestly covariant formulation of Hooke's law of elasticity.

It has been stressed by Phipps that the logical order of the development of physics is: first get kinematics right, and then go on to dynamics. Kinematics is defined as the science of pure motion, considered apart from causes. Now Phipps writes: "So defined, a kinematics of extended structures..."
that is (1) logically consistent and (2) of physical pedigree—i.e., capable of preserving the physical connectedness of stress and strain—does not currently exist."

Below I will show that the kinematics of special relativity is both logically consistent and of physical pedigree, in the sense of Phipps.

2. IS THE LORENTZ CONTRACTION UNIVERSAL?

Central in the discussion of Phipps is the relativistic concept "Lorentz contraction." Phipps writes: "With the advent of the Ehrenfest paradox it became apparent on logical grounds that the Lorentz contraction could not occur universally." This is shown by the following consideration: Where a circular disk of solid material is set into rotation about its center (considered at rest in an inertial frame $S$) the Lorentz transformation, applied locally to each portion of the disk, requires that (a) lengths transverse to the relative motion (as measured in $S$) transform invariantly, so the disk radius at all times retains a constant length in $S$, and (b) lengths parallel to relative motion contract, so that the disk rim should begin to contract in $S$ upon the onset of rotation. If the material of the disk is ideally uniform, there is in principle no weakest place where its structural integrity can first fail in consequence of such alleged azimuthal shrinkage. Hence a Lorentz contraction of the rime cannot occur in $S$—ergo the Lorentz contraction is not universal."

In the special theory of relativity the term "Lorentz contraction" has the following meaning. Let an object be at rest in the inertial frame $S'$. The frame $S$ moves with velocity $v$ relative to $S'$. If the distance in the $v$ direction between two points on the object, as measured in $S'$, is $l'$, then the distance between the two points, as measured by simultaneity in $S$, is $l = l'(1 - v^2/c^2)^{1/2}$. That $l < l'$ is referred to as the Lorentz contraction.

The definitional part here is the specification that the length of a body is measured by simultaneity in the reference frame of the observer. Then it follows universally, as a consequence of the postulates of special relativity, that $l$ is Lorentz contracted relatively to $l'$. In this connection an important observation was already made by Planck in 1910. He writes$^{(4)}$: "Der Satz, dass das Volumen eines mit der Geschwindigkeit $q$ bewegten Körpers einem ruhenden Beobachter im Verhältnis $(c^2 - q^2)^{1/2}$: $c$ kleiner erscheint als einem mit der Geschwindigkeit $q$ mitbewegten Beobachter, muss wohl unterschieden werden von dem anderen Satz, dass das Volumen eines Körpers sich im Verhältnis $(c^2 - q^2)^{1/2}$: $c$ verkleinert, wenn er von der Geschwindigkeit 0 auf die Geschwindigkeit $q$ gebracht wird. Ersterer Satz ist