GENERALIZED 'DEPENDENCIES' AND PARAMETER VARIANCE

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Abstract. The 'autodependence', (a special case of the – now quite obsolete – 'dependences', which had been introduced for very specialized astrometric purposes) is proportional to the parameter variance which is the expectation of the variance of the systematic error of a function evaluated with estimated parameters.

Key words: data adjustment, parameter variance, dependences

1. Reference Adjustments

Consider the relationship

\[ x = \varphi(\xi, a), \]

between \( x, \xi \) and \( a \). Let \( \xi \) be a vector of known independent variables and \( a^T = (a_1 \ a_2 \ \ldots \ a_n)^T \) be a vector of yet undetermined parameters. Without restricting generality, we may consider it to be linear, i.e.,

\[ x = \xi^T a. \] (1)

In order to find an estimate of \( a \), let values of \( x \) be observed for a sufficiently large number \( m \) of samples of vectors of variables \( \xi \) and collect them into the vector \( x_0^T = (x_{0\mu})^T \), so that we get from Eq. (1)

\[ \varepsilon_{\mu} + x_{\mu} = \sum_{\nu=1}^{n} \xi_{\mu \nu} a_{\nu}, \quad \mu \in \{1, \ldots, m\}, \] (2)

(where \( \varepsilon_{\mu} \) are the random observing errors) and in matrix form

\[ \varepsilon + x_0 = \Xi^T a. \] (3)

We call this a 'reference adjustment' because the Eqs. (1) and the condition equations (3), from which we estimate the parameters \( \hat{a} \), have the same form, in contrast to the more common situation where one uses \( a \), once estimated, to

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calculate some quantities $y$ that depend on independent variables $\eta$ having nothing to do with $\xi$ by $y = \psi(\eta, a)$.

It is then well known that the least-squares estimate $\hat{a}$ of $a$ is

$$\hat{a} = (\Xi \Xi^T)^{-1} \Xi \xi,$$

provided that the covariance matrix of $\varepsilon$ is the identity matrix $I$. In this case, $(\Xi \Xi^T)^{-1}$ is also the covariance matrix of $\hat{a}$.

Now consider a given vector $\xi$ of independent variables and calculate [from Eq. (1)] the (unobserved) corresponding $x$. Since the true $a$ is unavailable, we must use $\hat{a}$ as given by Eq. (4), so that we arrive at the estimate

$$\hat{x} = \xi^T (\Xi \Xi^T)^{-1} \Xi x_0$$

which is obviously a linear function of $x_0$. We introduce $D$, the vector of generalized dependences by

$$D^T = \xi^T (\Xi \Xi^T)^{-1} \Xi,$$

and thus have from (5) and (6)

$$\hat{x} = D^T x_0,$$

whence

$$\frac{\partial \hat{x}}{\partial x_{0\mu}} = D_\mu.$$

This shows that the proper way to get absolute parallaxes from 'relative' ones is by adding to them $\sum D_\mu \omega_\mu$ rather than $(1/m) \sum \omega_\mu$ ($\omega_\mu$ being the parallax of the $\mu$-th reference star), as some authors do. The dependence $D_\mu$ that is the factor of $x_{0\mu}$ – the $\mu$-th component of $x_0$ in Eq. (5) – is obviously

$$D_\mu = \xi^T (\Xi \Xi^T)^{-1} \xi_\mu.$$

Dependences were first introduced by Schlesinger (1911) for very simple forms of $\xi$ and later generalized, cf. e.g., Kiselev (1991).

2. The Parameter Variance

Each $\xi$ generates its own vector $D$ of dependences; these will thus have to be calculated separately for each $x$. This obviates their usefulness in general least-squares adjustment practice but leads, as Schlesinger recognized, to enormous savings in arithmetic effort – a consideration of prime importance at a time when