INTERACTION BETWEEN COPLANAR CRACKS UNDER DYNAMIC SHOCK LOADING

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Published data on stress concentrations in a three-dimensional body containing cracks under dynamic loading [1-4] have been obtained without allowance for interaction between the cracks. There are inertial effects due to crack interaction, which may have a considerable influence on the state of stress and strain in a body containing cracks, and ignoring them may lead to discrepancies between the limiting values of the external forces derived from theory and experiment.

Here we examine the interaction between coplanar cracks in an unbounded body in the presence of surface dynamic loading, which is described as a Heaviside time function or a Dirac delta function.

We consider an infinite elastic body that contains N cracks disposed in a single plane, which have smooth edges. We take local Cartesian coordinate systems Onx1nX2nx3n (n = i, N) such that the region Sn of crack n lies in the coordinate plane x1nx2n, and the opposite surfaces Sn of the cracks obey x3n = ±0.

When the surfaces of the cracks Sn (n = i, N) are subject to self-equilibrating dynamic forces and the crack contours are smooth, we can reduce the treatment of state of stress and strain that the body containing cracks has to a system of 3N integral equations of wave potential type. If only the normal external stresses Nn (n = i, N) are given at the surfaces of the cracks, this system is shortened to N equations and takes the form [5]

\[
\sum_{m=1}^{N} \int_{S_m} \left[ \frac{1}{2} \rho_m (\xi, t - t' \delta) \xi d\xi - 4 \rho_m (\xi, t - t' \delta) - (1 - 5\tau) \rho_m \times \right. \\
\left. \times (\xi, t - t' \delta) - 2 \tau \rho_m (\xi, t - t' \delta) + (1 - 2\tau) \tau \rho_m (\xi, t - t' \delta) \right] + \\
+ (\tau' - \tau^2 + 1/4) (t' \delta)^2 \rho_m (\xi, t - t' \delta) \left| x_{mn} - \xi \right|^2 dS = -N_n (x_n, t), (4G), \\
x_n \in S_n, t > 0, n = i, N, t'_\delta = |x_{mn} - \xi|/c_2.
\]

Here a prime denotes a derivative with respect to time; t is time; \( \gamma = c_2/c_1 \); \( c_1 \) and \( c_2 \) are the speeds of longitudinal and transverse waves in the body correspondingly; G is the shear modulus; \( x_n(x_{1n}, x_{2n}) \) is a point in \( S_n \) in the coordinate system linked to crack n; \( x_{mn}(x_{1mn}, x_{2mn}) \) the same point \( x_n \) in the coordinate system linked to crack m; \( B_n (n = i, N) \) unknown functions that are zero on the edges of the regions \( S_n \) correspondingly; and \( S_m(t) \) is the part of region \( S_m \) where \( t - t' \delta > 0 \).

The functions \( B_n (n = i, N) \) denote the displacements and stresses, where the relationship for the displacements implies that \( B_n \) represents a step in the opposite surfaces of crack n in the direction of the 0nx3n axis.

To transfer to the stationary integration region, we apply an integral Fourier transformation with respect to time to (1) and use the causality principle. System (1) becomes a Helmholtz-potential type in the Fourier transformation, which relates also to steady-state vibrations of a body containing cracks [6] and takes the form

\[
\sum_{n=1}^{N} \int_{S_n} \beta_n (\xi, s) R (\xi, x_{mn}, s) dS = N_n (x_n, s), (4G), x_n \in S_n, n = i, N.
\]
Here

\[ R(\xi, x_m, s) = [9 - 9i s_i |x_m - \xi| - (5 s_i^2 - s_i^4)|x_m - \xi|^2 + 4s_i^3 |x_m - \xi|^3 + 4s_i^4 |x_m - \xi|^4] |x_m - \xi|^2 - 4s_i^3 |x_m - \xi|^3 \exp i s_i |x_m - \xi|^4 \]

with \( s \) the parameter in the Fourier transformation, which gives the Fourier transforms from the individual functions.

If the cracks are disks with radii \( a_n \) \((n = 1, N)\), we seek the \( \beta_n(x_n, s) \) in the class of functions represented in the form

\[ \beta_n(x_n, s) = V a_n - x_n - x_n \alpha_n(x_n, s), \quad n = 1, N, \]  

in which \( a_n(x_n, s) \) are unknown but doubly differentiable functions.

We use (4) with formulas for the stresses [5] to get the stress intensity coefficient for the normal loading \( K_{1n} \) for crack in expressed in terms of \( \alpha_n \):

\[ K_{1n}(\psi, t) = - GV a_n (1 - \mu)^{-1} \int a_n(\psi, s) \exp(-ist) ds, \quad n = 1, N, \]

in which \( \psi_n \) is the angular coordinate of point \( x_n \), which is on the edge of a crack, and \( \mu \) is Poisson's ratio.

It is possible to transfer to a semiinfinite integration range in (5) for any form of loading. For example, if the normal forces that act on the surfaces of cracks with identical radius \( a \) vary in accordance with \( N_n = N_0 S_+ (t) \) \((n = 1, N)\), in which \( N_0 = \text{const} \), \( S_+ \) is a Heaviside function, and \( t = t c_2/(2a) \) is the relative time, then the relative stress intensity coefficients \( \tilde{K}_{1n} \) \((n = 1, N)\) are given by

\[ \tilde{K}_{1n}(\psi, \tilde{t}) = \pi \int \frac{\{\text{Im} [\tilde{a}_n(\psi, x)] \cos(2x \tilde{t}) - \text{Re} [\tilde{a}_n(\psi, x)] \sin(2x \tilde{t})\} x^{-1} dx. \]

Here \( \tilde{K}_{1n} = K_{1n}/K_\alpha \) \((K_\alpha = 2N_0 \sqrt{a}/\pi)\) is the static stress intensity coefficient for the normal loading for an isolated crack under force \( N_0 \), and \( \alpha_n(n = 1, N) \) are solutions to (2) with the variable \( s \) replaced by \( \kappa c_2/a \), with identical right sides and identical radii for the cracks.

If the force variation at the cracks is described by \( N_n = N_0 \delta_+(t) \) \((n = 1, N)\), where \( \delta_+ \) is a Dirac delta function, (5) can be put as

\[ \tilde{K}_{1n}(\psi, \tilde{t}) = -\pi \int \frac{\text{Re} [\tilde{a}_n(\psi, x)] \cos(2x \tilde{t}) + \text{Im} [\tilde{a}_n(\psi, x)] \sin(2x \tilde{t}) dx, \]

in which \( \tilde{K}_{1n} = K_{1n}/K_\alpha \).