Modified Weyl Theory and Extended Elementary Objects

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To represent extension of objects in particle physics, a modified Weyl theory is used by gauging the curvature radius of the local fibers in a soldered bundle over space-time possessing a homogeneous space $G/H$ of the $(4, 1)$-de Sitter group $G$ as fiber. Objects with extension determined by a fundamental length parameter $R_0$ appear as islands $D(i)$ in space-time characterized by a geometry of the Cartan-Weyl type (i.e., involving torsion and modified Weyl degrees of freedom). Farther away from the domains $D(i)$, space-time is identified with the pseudo-Riemannian space of general relativity. Extension and symmetry breaking are described by a set of additional fields $(\xi^a(x), R(x)) \in G/H \times R^+$, given as a section on an associated bundle $\tilde{E}(B, G/H \times R^+, \tilde{G})$ over space-time $B$ with structural group $\tilde{G} = G \otimes D(1)$, where $D(1)$ is the dilation group. Field equations for the quantities defining the underlying bundle geometry and for the fields $\xi^a(x)$ are established involving matter source currents derived from a generalized spinor wave function. Einstein's equations for the metric are regarded as the part of the $\tilde{G}$-gauge theory related to the Lorentz subgroup $H$ of $G$ exhibiting thereby the broken nature of the $\tilde{G}$-symmetry for regions outside the domains $D(i)$.

1. INTRODUCTION

According to common thinking, the words “elementary” and “extended” refer to notions which are excluding each other: Elementary objects in physics are usually considered to be pointlike, i.e., not extended; while extended objects are regarded to be composed of subunits or constituents

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(partons, quarks, preons, etc.) the orbital motion of which generates the
extension of the composite objects; hence they are not elementary. In this
article I would like to present a— one could say—holistic description of
elementary yet extended objects in particle physics which possesses one or
several gauge interactions. The aim is to ultimately understand, within the
framework of gauge theories, the spectrum of hadrons, i.e., proton,
neutron, lambda, pi, kay, etc., which are elementary states of the strong
interaction being all extended objects.

The magic word in this context is “geometry.” Whatever happens at
small distances (i.e., in high-energy collisions) there must be a definite
geometry realized in nature the knowledge of which is crucial in formu-
lation of a dynamics for interactions at small distances. This geometry may
be the four-dimensional flat Minkowski space-time geometry of special
relativity as usually assumed in field theory; or it may be a higher-
dimensional geometry involving curvature, torsion, and possibly further
geometric properties yielding a “compactified” \((4 + N)\)-dimensional space
involving, besides the four space-time coordinates, further \(N\) internal
coordinates; or it may be what some physicists would call a quantum
geometry, i.e., a geometry showing quantum or stochastic fluctuations at
small distances. Whatever particular configuration space geometry is
suggested or assumed in the context of high-energy physics, I think it is
wise to remember that there is, indeed, a definite geometry realized in the
small in our surrounding world, and that this geometry may be different
from what we observe at large distances. I might mention in this context
that the problem of determining the geometry realized in nature at
small distances was already noted by Bernhard Riemann in his famous
Habilitationsschrift of 1867.\(^{(1)}\) He pointed out that this geometry has to be
established by empirical means and cannot be decided upon \emph{a priori}.

The task of the physicist thus is to determine the geometry realized in
the small—or to guess it if a straightforward determination with the help
of experiments turns out to be impossible. Guesswork, of course, is not
enough: One would have to work out all the consequences of an educated
guess and compare them with what is observed in nature in order to see
whether the original hypothesis concerning the geometry realized in the
small is indeed correct.

Before I start formulating a dynamics which has a chance of having
anything to do with strong interactions and extended particles, let me thus
first talk about a particular geometric stratum—a certain geometry
provided by a fiber bundle raised over space-time as a base—intended to
reflect certain features which are regarded as essential in a geometric
formulation of a gauge dynamics at small distances. We shall encounter
\emph{three} types of objects: