Field Theory on $R \times S^3$ Topology: Lagrangian Formulation

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Received November 8, 1988

A brief description of the ordinary field theory, from the variational and Noether's theorem point of view, is outlined. A discussion is then given of the field equations of Klein-Gordon, Schrödinger, Dirac, Weyl, and Maxwell in their ordinary form on the Minkowskian space-time manifold as well as on the topological space-time manifold $R \times S^3$ as they were formulated by Carmeli and Malin, including the latter's most general solutions. We then formulate the general variational principle in the $R \times S^3$ topological space, from which we derive the field equations in this space.

1. INTRODUCTION

Recently, Carmeli and Malin\(^{(1-6)}\) formulated a field theory on $R \times S^3$ topology, i.e., the space of time and rotations. This approach is significant in those problems in which only the angular dependence is relevant rather than the distance, as in the Minkowskian spacetime.

For instance, let two particles with magnetic moments $\mu$ and $\mu_1$ be located at the origin of coordinates, with the angle $\phi$ between the magnetic moments. The torque active between them is only a function of the angle $\phi$ between the magnetic moments. We can write the torque as follows:

$$N = \mu \mu_1 \cos \phi /2$$

Indeed, this torque, based on our daily experience, can easily be checked, as can be seen from Fig. 1.

For the values of $\phi$, it can immediately be seen that the description of

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the torque field, given by Eq. (1.1), describes correctly the actual field. Thus the problem can be solved in rotational topology with no reference to distance.

In Section 2, we briefly review field theory in its ordinary formulation in Minkowskian space-time (Klein-Gordon, Schrödinger, Dirac, Weyl, and electromagnetic). In Section 3, we describe the variational principle in Minkowskian spacetime, along with Noether's theorem. In Section 4 a summary of the field theory on $R \times S^3$ topology according to Carmeli and Malin is given, along with the solutions of the field equations in this space. In Section 5 we formulate a general variational principle on this topology, writing the Lagrangians for these fields, and derive from them the field equations in $R \times S^3$ topology. Section 6 is devoted to the summary and concluding remarks.

2. FIELD THEORY ON MINKOWSKIAN SPACE-TIME

Before we describe the field theory on $R \times S^3$ we will give a short review of the field theory on ordinary space-time.

2.1. The Schrödinger Equation

As is well known, the Schrödinger equation is the base for quantum mechanics. One of the most important axioms of this theory is that each classical dynamical quantity is replaced by an operator. In this way, we