Field Theory on $R \times S^3$ Topology. VI: Gravitation

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We extend to curved space-time the field theory on $R \times S^3$ topology in which field equations were obtained for scalar particles, spin one-half particles, the electromagnetic field of magnetic moments, an $SU_2$ gauge theory, and a Schrödinger-type equation, as compared to ordinary field equations that are formulated on a Minkowskian metric. The theory obtained is an angular-momentum representation of gravitation. Gravitational field equations are presented and compared to the Einstein field equations, and the mathematical and physical similarity and differences between them are pointed out. The problem of motion is discussed, and the equations of motion of a rigid body are developed and given explicitly. One result which is worth emphasizing is that while general relativity theory yields Newton's law of motion in the lowest approximation, our theory gives Euler's equations of motion for a rigid body in its lowest approximation.

1. INTRODUCTION

This paper is a continuation of the efforts to describe physical laws in terms of rotations and thus with $R \times S^3$ invariance rather than the ordinary description in the Minkowskian space-time.$^{(1-5)}$ Here $R$ describes the real line of the time coordinate and $S^3$ the 3-sphere of the rotational coordinates usually described by the three Euler angles. In the $R \times S^3$ topology the angles are coupled to time just as in special relativity the distances are coupled to the time coordinate.$^{(6,7)}$ Angles and time mix up through a four-dimensional transformation of the Lorentz type, just as distances and time mix up through the ordinary Lorentz transformation. Angular velocities,
angular momenta, and moment of inertia subsequently replace the ordinary linear velocities, linear momenta, and mass that occur in special relativity. In this paper we extend to curved space-time the concepts of field theory on $R \times S^3$ topology.

In Sec. 2 we briefly review the foundations of the ordinary general relativity theory, and in Sec. 3 we give the preliminary concepts needed from the $R \times S^3$ topology field theory so as to establish our conventions and notations. In Sec. 4 we make the extension of the $R \times S^3$ field theory to curved space-time. The theory obtained is essentially an angular-momentum representation of gravitation. The gravitational field equations are given and compared with the ordinary Einstein field equations. The problem of motion, an extremely important topic in any theory of gravitation, is subsequently discussed in Sec. 5, and the equations of motion for a rigid body are derived and given explicitly. It is shown that, while in general relativity theory we are accustomed to having Newton’s second law of motion in the lowest approximation, our theory yields Euler’s equations of motion for a rigid body in the lowest approximation. The last section is devoted to the concluding remarks. Throughout this paper the signature is taken as $(+ - - -)$.

2. GENERAL RELATIVITY THEORY: A BRIEF REVIEW

In this section we briefly review the foundations of the general relativity theory so as to establish our notation. For more on this theory the reader is referred to the standard textbooks.\textsuperscript{8-11}

The physical foundations of general relativity theory are based on two principles: the principle of equivalence and the principle of general covariance.

The first can be stated as the assumption that in a freely falling, non-rotating, laboratory the local laws of physics take on some standard form including a standard numerical content independent of the position of the laboratory in space. It is implicit in this statement that the effects of gradients in the gravitational field strength are negligibly small, namely the tidal interaction effects are negligible. A weaker version for the equivalence principle states only that the local gravitational acceleration is substantially independent of the composition and structure of the matter being accelerated.

According to the principle of general covariance, the coordinates are nothing more than a bookkeeping system to label the events. The principle is a valuable guide to deducing correct equations of physics and is often stated in one of the following (not equivalent) ways: