Acoustic emission in brittle solids

W. E. SWINDLEHURST
Atomenergikommissionens Forsøgsanlæg Risø, 4000 Roskilde, Denmark

T. R. WILSHAW
Effects Technology, Inc, 5383 Hollister Avenue, Santa Barbara, California, USA

A signal/source correlation study of the stress waves emitted during unstable microscopic Hertzian fracture in glass is described. A theoretical analysis of the variation in excess strain energy with applied load is made and the results compared with experimental data covering a wide range of crack sizes.

1. Introduction
An essential step in the development of the acoustic emission detection technique, both for non-destructive testing applications and as a materials science research tool, is the establishment of a signal/source correlation. Accordingly, in recent years a number of studies [1–14] have been made on a wide variety of materials in an attempt to relate recorded emission data to their source characteristics. However, in all such experiments, a major problem is the choice of material and deformation mode for which the emission source can be readily identified and quantitatively analysed. As a part of a continuing effort in this direction, the investigation presented in this paper is directed towards an understanding of the brittle fracture process and its associated acoustic emission. However, with the above mentioned difficulties in mind, the present experiments were designed using the indentation fracture phenomenon [5] as a controlled microscopic emission source which is amenable to fracture mechanics modelling.

The particular type of indentation fracture used is that caused by the loading of a sphere normally onto a brittle solid surface. Both the stress field and the deformation associated with this indentation geometry are generally known as Hertzian, after Hertz [6] who originally formulated the stress field solution. At a critical stage during the loading of such an indenter, a conically shaped crack (Fig. 1) may be suddenly produced in the region of the contact circle. The fracture is caused by the extension of an already present surface flaw in the highly inhomogeneous subsurface stress field. A further advantage in using this indentation system is that it is being current employed in the study of crack growth in brittle materials [7–10]. In this application, the load at which the crack rapidly develops is used to quantify the fracture behaviour of the material under investigation. Consequently, the ability to detect crack growth acoustically will find direct application in such investigations.

2. A model for emission behaviour
A method is described for calculating the total excess energy, \( \Delta U_k \), which is generated during the growth of a cone crack in a Hertzian stress field created in an ideally brittle solid. This energy comprises the total mechanical energy released during crack growth minus that necessarily re-
quired to propagate the crack. From a mechanistic point of view, ΔU_k will manifest itself as the kinetic energy of the rapidly separating crack walls. A fraction of this energy will then be transmitted into the bulk of the solid in the form of stress waves—detectable as acoustic emission. ΔU_k represents the maximum energy available for conversion into acoustic emission. No attempt is made to calculate the exact fraction so converted.

The system to be studied is an ideally brittle body of unit thickness which is subject to external forces at its boundaries [11]. On the Griffith model, a crack in such a material is assumed to be in equilibrium if \( dU_T/dc = 0 \), where \( U_T \) is the total energy of the system and \( c \) the crack length. \( dU_T/dc \) considered to be composed of two terms, one involving \( G \), the rate of change of mechanical energy with crack length and the other \( \gamma \), the specific surface energy of the body

\[
dU_T/dc = -G + 2\gamma.
\] (1)

If, in addition, \( d^2U_T/dc^2 \) is positive, the system is in unstable equilibrium and a finite increase \( \Delta c \) in the crack length results in a decrease in \( U_T \).

\[
\Delta U_T = -G\Delta c + 2\gamma\Delta c < 0.
\] (2)

Thus, excess energy, \( \Delta U_k \), is released beyond that required for crack extension. To account for the presence of this energy in the system we may write

\[
-G\Delta c + 2\gamma\Delta c + \Delta U_k = 0.
\] (3)

Accordingly, this excess energy released during crack growth from \( c_1 \) to \( c_2 \) may be written as

\[
\Delta U_k = \int_{c_1}^{c_2} Gdc - 2\int_{c_1}^{c_2} \gamma dc
\] (4)

where \( \Delta U_k \) has the units of energy per unit thickness. To include the possibility of a variable crack front width, \( L \), the above equation becomes

\[
\Delta U_k = \int_{c_1}^{c_2} GLdc - G_c \int_{c_1}^{c_2} L dc
\] (5)

where \( G_c = 2\gamma \) and \( \Delta U_k \) has now absolute units of energy. This equation is generally applicable to unstable fracture situations in brittle materials. Specific forms of the equation have been described in the literature using expressions for \( G \) and \( L \) appropriate to the particular testing configurations employed, for example Gerberich and Hartbower [12] use \( G = \pi \sigma^2 c/E \) (where \( \sigma \) is the tensile stress on the specimen and \( E \) Young's modulus) and a constant \( L \). Under these conditions the first term in Equation 5 can be easily evaluated analytically.

However, because of the highly inhomogeneous nature of the Hertzian stress field, \( G \) in this case has to be analysed numerically as a function of \( c \). This calculation has been performed by Lawn [13] and Frank and Lawn [14] for crack growth initiating from the edge of the contact circle. An extension of this analysis has been made by Wilshaw [7] for cracks initiating beyond the edge of the contact circle edge. In all these analyses, \( G \) is usually represented as a function of \( c \) in normalized co-ordinates, \(-G/G_c \) as a function of \( c/a \) (the instantaneous crack length normalized to the instantaneous value of the contact circle radius, \( a \)). An example of such a diagram, after Wilshaw [7] is given in Fig. 2a.

These curves can be used to provide a semi-quantitative insight into the relative amounts of excess energy released during cone growth under different conditions. For unit width of crack front, Equation 5 can be written as

\[
\Delta U_k/G_c = \int_{(c/a)_1}^{(c/a)_2} a(G/G_c)dc - \int_{(c/a)_1}^{(c/a)_2} ad(c/a)
\] (6)

Figure 2 The normalized strain energy release rate, \( G/G_c \), as a function of the normalized crack length, \( c/a \), for Hertzian cone fracture.