Stochastic Microcausality in Relativistic Quantum Mechanics

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Received June 17, 1983

A recently formulated concept of stochastic localizability is shown to be consistent with a concept of stochastic microcausality, which avoids the conclusions of Hegerfeldt's no-go theorem as to the inconsistency of sharp localizability of quantum particles and Einstein causality. The proposed localizability on quantum space-time is shown to lead to strict asymptotic causality. For finite time evolutions, upper bounds on propagation to the exterior of stochastic light cones are derived which show that the resulting probabilities are too small to be actually observable in a realistic context.

1. INTRODUCTION

A systematic analysis of the foundations of relativity and of quantum theory at the historical as well as the epistemological level reveals that these foundations reside in measurement-theoretical definitions of such basic concepts as space-time events and their separations as well as of particle observables such as position and momentum. The fact that the foundations of relativity rest on operational definitions, but that the implicitly assumed properties of these definitions are irreconcilable with quantum mechanics in general, and the uncertainty principle in particular was pointed out by Einstein in print a number of times, and especially in a 1949 "reply to criticisms", as he discussed the rationale behind attempting to extrapolate these definitions from the macroscopic domain to the microscopic world of atomic and subatomic dimensions. The key point is that the very definition of space-time separation (and therefore the physical meaning of metric

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2 Supported by an NSERC Fellowship.
3 Supported in part by NSERC research grant No. A5206.
tensor) rests on the possibility of exchanging "arbitrarily sharp optical signals" (Ref. 2, p. 676) between point test particles on the one hand, and that, on the other hand, the employment of such signals destroys all knowledge of test particle localization. The introduction of compensatory countersignals cannot rescue the situation if the uncertainty principle is valid, since the effectiveness of such countersignals rests on the possibility of knowing with arbitrary accuracy the simultaneous position and momentum of the test particles in question.

Thus, the issue of particle localization lies at the very foundations of relativistic quantum theory. In view of the aforementioned physical inconsistency of any theory incorporating both the uncertainty principle and the presupposed feasibility of point localization in the relativistic context, it is not surprising that none of the many past attempts (cf. Sections 2.1 and 2.2 of Ref. 1 for a review and references) at arriving at a cogent concept of point locality in relativistic quantum mechanics have met with true success. In recent years it became clear to a number of researchers (3-9) that implicit in the foundations of conventional relativistic quantum theory there are mathematical inconsistencies that reflect physical inconsistencies. The most striking of these inconsistencies is sometimes referred to (5) as Hegerfeldt's theorem (4), and it is a statement to the effect that any notion of particle localizability that allows for the possibility of states that are localized with total certainty in any bounded region of space-time, or a bounded region of space at a given time (i.e., sharply localized), is mathematically incompatible with Einstein causality. The essence of the most recent proofs (8) of this theorem lies in showing that even when using the weakest conditions under which the notion of localizability still makes sense, any spatially sharply localized wave packet will spread instantaneously to any other region of space.

The stochastic quantum mechanics approach (1) developed in recent years rejects the concept of sharp quantum localizability as being operationally unfeasible, physically inconsistent, and mathematically too restrictive, and replaces it with that of stochastic localizability at the nonrelativistic as well as the relativistic level. As a result, a physically and mathematically consistent framework for relativistic quantum mechanics emerges, which appears capable of dealing also with other unsatisfactory aspects of the conventional approach to this subject. (1) In Section 2 we shall discuss some of the basic physical and mathematical features of this formalism in the context of localizability of relativistic quantum particles. In Section 3 we then show how the ensuing concept of stochastic localizability gives rise to a concept of stochastic microcausality. In Section 4 we prove that the free propagation of (spin-zero) stochastically extended particles is asymptotically causal, and therefore causally indistinguishable, in the context