DIFFUSION BURNING OF A LIQUID-FUEL DROP IN A TWO-OXIDIZER MIXTURE

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The diffusion theory of combustion of a drop of liquid fuel proposed in a simplified variant by G. A. Varshavskii [1] (i.e., on the assumption of spherical symmetry and transport coefficients independent of temperature and the local composition of the mixture) leads to the following relation for the evaporation rate of the burning drop $g$:

$$
\frac{q}{\pi r_0 D} \left( \frac{1}{r_0} - \frac{1}{r_1} \right) = \ln |B|, \quad (1)
$$

where

$$
B = 1 - \frac{\varepsilon r}{l} (T_0 - T_3) + \frac{\eta \rho_0}{l} \sum q_i n_i. \quad (2)
$$

This relation was obtained for the case when the gas surrounding the drop contains only one oxidizer (for example, oxygen). Actually, the gas medium may contain several such oxidizers. It is easiest to extend the calculations to the case when none of the oxidizers reacts with any of the products (final or intermediate) of the reactions between the fuel and the other oxidizers. In this case Eq. (1) holds, but Eq. (2) must be replaced by

$$
B = 1 - \frac{\varepsilon r}{l} (T_0 - T_3) + \frac{1}{l} \sum q_i n_i. \quad (3)
$$

Less obvious is the result when the products of the reaction of the fuel and one of the oxidizers may react with another oxidizer. Below we consider the case of two oxidizers: oxygen and water vapor. Naturally, this choice is not obligatory and has been made only for the sake of determinacy.

At first glance it appears that the water vapor does not take part in the reaction at all until the oxygen has been entirely consumed. In fact, owing to the diffusional nature of the process, the hydrogen produced in the oxidation reaction between the fuel and the water vapor is, in its turn, oxidized by the oxygen. Thus, the consumption of water vapor is equal to zero. This is correct, but it does not imply that the water vapor may be regarded as an inert gas and the rate of evaporation calculated from (1) and (2).

A spherical drop of fuel of radius $r_0$ (see Fig. 1) at temperature $T_0$ is surrounded by a "reduced film" of radius $r_2$. The temperature of the medium $T_2$ and the mass fractions of oxygen $n_0$ and water vapor $n_w$ are given. It is assumed that the Lewis number $L = 1$, the product of the diffusion coefficient, and the density of the mixture is the same for all the components and does not depend on the coordinate $D = \text{const}$. The
specific heat of the mixture does not depend on the coordinate $c_p = \text{const.}$* It is assumed that the process is completely diffusional — reaction is possible only at the zone boundaries ($r_1$ and $r_2$); the concentrations of the corresponding components at these boundaries are vanishingly small; two reacting components cannot be present anywhere at the same time. The process is stationary, i.e., all the heat supplied to the drop is expended on evaporation; matter and enthalpy are nowhere accumulated.

The total mass flux through an arbitrary sphere of radius $r$ is equal to the flow of fuel $g$, the rate of evaporation of the fuel drop. The mass flux of any component in any zone

$$
\mathbf{g}_i = -4 \pi D D \frac{dn_j}{dr} + gn_j.
$$

Here, $i = 1, 2, 3$ is the zone number; $j = O, W, T$ is the component index. The components also include the "temperature" $n_T = T$, $g_{T1} = Q_1/c_p$. For example, $g_{O3}$ is the oxygen flux in the third zone, $g_{T2}$ is the fraction obtained by dividing the heat flux in the second zone by the specific heat.

Integration of (4) over any zone gives

$$
\frac{g}{4 \pi D} \left( \frac{1}{r_{i-1}^2} - \frac{1}{r_i^2} \right) = \ln |B_i|.
$$

Here, because the left side of Eq. (5) are the same,

$$
B_i = \frac{g_{j1} - gn_{j1}}{g_{j1} - gn_{j1,i-1}}
$$

is the same for all the components, i.e., depends only on the zone number $i$ and not on the component index $j$.

The object of the subsequent computations is to reduce Eqs. (5) and (6) to a form analogous to that of (1) and (2), i.e., to eliminate the zone radii $r_1, r_2$, and the corresponding values of the temperature and concentrations. The zone radii are eliminated by simple addition

$$
\frac{g}{4 \pi D} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) = \ln (B_1, B_2, B_3).
$$

Expressions for all the $B_i$ are obtained after substituting the boundary values of the concentrations and temperature and taking into account the stoichiometric relations

$$
B_1 = \frac{t_1 - T_1}{t_1 - T_0}
$$

(temperature, first zone);

$$
B_2 = \frac{t_2 - T_2}{t_2 - T_1} = 1 + \frac{n_{W2}}{\beta_1}
$$

(temperature and water vapor, second zone);

$$
B_3 = \frac{t_3 - T_3}{t_3 - T_2} = 1 + \frac{n_{O2}}{\beta_1} = \frac{n_{W2}}{\beta_1}
$$

(temperature, oxygen and water vapor, third zone).

Here, we have introduced the notation $t_i = g_{T1}/g$; $\beta_1, \beta_2$ are the "water/fuel" and "oxygen/fuel" mass stoichiometric coefficients, respectively.

From (9) and (10), eliminating $n_{W2}$, we easily find

$$
B_2 B_3 = 1 + \frac{n_{W2}}{\beta_1} + \frac{n_{O2}}{\beta_2}
$$

Using Eqs. (9) and (10) to formulate an expression for the temperature at the outer boundary of the first zone $T_1$ and substituting it in (8), we obtain

*As a consequence of these assumptions the thermal conductivity of the gas mixture does not depend on the coordinate $\lambda = \text{const.}$.