OPTIMIZATION OF THE AMPLIFICATION COEFFICIENT
IN THE CARBON DIOXIDE GASDYNAMIC LASER

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The present study will offer results of a numerical solution of the problem of optimization to obtain the maximum optical amplification coefficient in a mixture containing CO\textsubscript{2}, N\textsubscript{2}, He, and H\textsubscript{2}O molecules. The relaxing gas flow in the nozzle is considered one-dimensional, and the effect of gas viscosity is not considered.

To describe the kinetics of vibrational energy exchange the system employed in [1, 2] is used. According to [3], values of the quantities $Q_{23}^{(l)}$ appearing in the expression for probability of energy exchange between the second and third types of carbon dioxide molecule vibrations $Q_{23}^{(l)} = \xi_{CO_2}Q_{23}^{(l)} + \xi_{N_2}Q_{23}^{(l)} + \xi_{He}Q_{23}^{(l)} + \xi_{H_2O}Q_{23}^{(l)}$ (where $\xi$ is the molar fraction of component) may be written in the form $Q_{23}^{(l)} = \exp(a_{23}^{(l)}x^3 + a_{23}^{(l)}x^2 + a_{23}^{(l)}x + a_{23}^{(l)})$; while according to [4, 5] the form $Q_{23}^{(l)} = 100e^{a_{23}^{(l)}x^3 + a_{23}^{(l)}x^2 + a_{23}^{(l)}x + a_{23}^{(l)}}$ may be used, where $x = T^{-1/3}$ and $T$ is the translational temperature of the gas. The constants appearing in the $Q_{23}^{(l)}$ expressions are presented in Table 1. The possibility of using both approximations will be considered.

We consider the class of nozzles with linear change in the ratio of supersonic nozzle segment area $A(x)$ to critical section $A_\ast$; i.e., $A(x)/A_\ast = 1 + ax$ (where $x$ is the distance from the critical section). Optimization was performed with respect to the initial conditions and parameters $a$ and $L$ (see [2]). The search was performed by the configurations method, the fundamentals of which are described in [6].

Work [7] presented the variation problem of search for a maximum in the amplification coefficient of a mixture of CO\textsubscript{2} + N\textsubscript{2} + He over a narrow parameter range. With respect to that study, we note that for a simultaneous flow, the solution of the problem of flow of the relaxing gas in the nozzle depends solely on $A(x)/A_\ast$ (dependence on $A_\ast$ appears only in the two-dimensional approximation), so that simultaneous search for $A(x)$ and $A_\ast$ is incorrect. It is obvious that in the one-dimensional approximation there is no point in separate searches for the optimum in the cases of axisymmetric and planar nozzles, since one and the same solution will be obtained for the optimum regime in each case.

Figures 1-4, 6 present results of solution of the optimization problem for various compositions as a function of the parameter $a$, which characterizes nozzle geometry (on all graphs $\xi$ is expressed in molar fractions). It is evident from the results presented that at high values of $a$, high values of optimum temperature $T_\text{opt}$ are necessary. This leads to disruption of the condition of absence of chemical transformations. In reality, upon increase in temperature the CO\textsubscript{2} molecules decompose first (in Fig. 5 the solid curves correspond to equilibrium concentrations of molar fractions of molecules CO\textsubscript{2}, CO, and O\textsubscript{2} formed by dissociation at corresponding optimum values of $T_\text{opt}$ and $p_\text{opt}$ for a CO\textsubscript{2} + N\textsubscript{2} + He mixture). Therefore, the question arises of how markedly the value of $k$ falls upon decrease in $T_\text{opt}$. Some idea of this is given by the dashed-dot lines of Figs. 4 and 6, corresponding to conditions producing a $k$ value $5\%$ less than optimum. The results obtained indicate that the relief of the multidimensional surface near the optimum point is smooth (see also the results of [2]) so that the temperature difference reaches $600^\circ$K.

It was assumed that the value of the collision half-width appearing in the expression for optical amplification coefficient $A = 1/\sqrt{T}$, and that the value of the Einstein coefficient $A_{mn} = 0.21$, after [6]. Measurements [9] have shown that $A_{mn}$ is $20\%$ less than the value taken here. The quantity $A_{mn}$ appears linearly in the expression for the optical amplification coefficient, so that refinement of $A_{mn}$ does not affect the character of optimization, merely changing the absolute value of $k$ proportionally. In the approximation of $Q_{23}^{(l)}$ [4, 5] were used in all variants.


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Fig. 1. Optimum values of parameters $T_0$, $p_0$, $\xi_{CO2}$, $\xi_{He}$, $L$, and maximum $k$ value as functions of $a$, mixture CO$_2$+He (here and in Figs. 2, 3, and 6 values of $Q_{23}^{(i)}$ ($i$ = CO$_2$, N$_2$, He) are taken from [3]).

Fig. 2. Solution of optimization problem for mixture CO$_2$+N$_2$.

Fig. 3. Solution of optimization problem for mixture CO$_2$+N$_2$+H$_2$O.

Fig. 4. Solution of optimization problem for mixture CO$_2$+N$_2$+H$_2$O. $\longrightarrow$ $Q_{23}^{(i)}$ [3]; $\longrightarrow$ $Q_{23}^{(i)}$ [4]; $\longrightarrow$ values of initial temperature $T_0$ at which $k$ is 5% less than optimum.

Fig. 5. Equilibrium values of molar fractions of CO$_2$, CO, and O$_2$ molecules formed by dissociation; $\longrightarrow$ equilibrium values at optimum temperature $T_0$ ($a$ = $\xi_{CO2}$, value from optimization problem); $\longrightarrow$ at temperature $T_0$. 