ON THE THEORY OF INERTIAL GRAVITATIONAL FIELDS

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ABSTRACT

The generalized equations of the inertial gravitational field are derived from variational principles. It is shown that variational properties of inertial gravitational potentials have important peculiarities which cause peculiarities of equations obtained.

The equations of free motion of test particles in curved space-time (in the gravitational field) are the geodesic equations

\[ \mu \frac{Du^i}{ds} = \mu \frac{du^i}{ds} + \mu \Gamma^i_{kl}u^ku^l = 0, \]

where

\[ u^i = \frac{dx^i}{ds} \]

are components of the 4-velocity of test particles; \( ds \) is an element of the space-time interval:

\[ ds^2 = -g_{ik}dx^idx^k; \]

\( D/ds \) is the operator of the absolute derivative with respect to \( s \); \( g_{ik} \) is the metric tensor; \( \mu = \hat{\mu}c^2 \); \( \hat{\mu} \) is the proper mass density of the dust-like matter consisting of test particles; \( \Gamma^i_{kl} \) are Christoffel symbols.

In equations (1) \( \Gamma^i_{kl} \) play the role of the 'dynamical characteristics' ('strengths') of the gravitational field (see, for example, [1]) and they are used to form the gravitational Lagrangian from which Einstein's equations can be derived on the basis of variational principles.
tional principles:

\[ R_{ik} - \frac{1}{2} g_{ik} R = \frac{2}{\alpha} T_{ik}, \]  

(4)

where \( R_{ik} \) is the Ricci tensor, \( R = g^{ik} R_{ik} \), \( \alpha = \sigma^4/mk \), \( k \) is Newton's constant of gravitation, \( T_{ik} \) is the energy-momentum tensor of the gravitating matter.

The term \( du_i/ds \) (in equations (1)) in the interpretation of geodesic equations given above is not considered as the 'field quantity', that is why it is not included in the theory of the gravitational field directly.

Therefore we suggest that another interpretation of geodesic equations should be examined.

Taking into account that

\[ u_{ki} u^k = g_{ik} u^i u^k = -1, \]  

\[ u_{ki} u^k = 0, \]  

and

\[ \frac{Du_i}{ds} = u_{i; k} u^k, \]  

we have

\[ u_G_{ki} u^k = 0, \]  

(1a)

where

\[ G_{ki} = u_{i; k} - u_{k; i} = u_{i, k} - u_{k, i}, \]  

(6)

(a semicolon and a comma denote the covariant and the partial derivatives respectively).

Equations (1a) are geodesic equations of the 'second kind' and can be transformed into the standard geodesic equations (1) directly.

Equations (1a) are equations of free motion of test particles in the space-time as well as equations (1); they can be obtained from variational principles:

\[ \delta S = 0 \]  

(7)

where

\[ S = -\frac{1}{\alpha} \int ds, \]