Rapid Communication

Excitations bound to a vortex line in layered superconductors

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Using Bogoliubov-de Gennes equation, we study quasiparticle excitations in layered superconductors in the presence of a straight vortex line which is parallel to the layers. The lowest bound state is shown to have energy eigenvalue of the order of magnitude $\Delta$, the energy gap, in contrast to the corresponding value $\Delta^2/\hbar v_F$ when the line is perpendicular to the layers.

1. Quasiparticle excitations in type 2 superconductors in the presence of a vortex line were first studied by Caroli, de Gennes and Matricon. The most interesting result is that there exist quasiparticle states bound to the vortex line with energy smaller than the energy gap. Recently, Hess et al. obtained remarkable images of the vortex cores in NbSe$_2$ by a low temperature scanning-tunneling-microscope (STM). Subsequently, solving Bogoliubov-de Gennes (B-G) equation numerically, Shore et al. and Gygi and Schluter have shown that the images are due to these low-lying excitations. It is worth pointing out that in Abrikosov states these excitations form bands due to quantum tunneling of the bound quasiparticles between adjacent lines and possibly contribute to the Haas-van Alphen effect.

The purpose of the present letter is to point out that in layered superconductors such as copper oxide compounds the bound excitations have quite different energy spectrum when the vortex line is parallel to the layers. We use a model where the pairing interaction acts only within the layers and interlayer coupling is provided only through electron transfer, and solve corresponding B-G equation numerically.

2. As the pair potential in B-G equation, we use solutions of the Ginzburg-Landau (GL) equation for the Lawrence-Doniach model of layered superconductors. We write $\psi$ in dimensionless form as

$$
\frac{\partial F}{\partial \psi^*(x,j)} = -\frac{d^2}{dz^2} \psi(x,j) - \frac{\eta}{2^2} \left( \psi(x,j + 1) + \psi(x,j - 1) - 2\psi(x,j) \right)
$$

$$
-\psi(x,j) + |\psi(x,j)|^2 \psi(x,j) = 0,
$$

(1)

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where \( j \) labels the layers and \( x \) the coordinate in each layer, hence \( \psi(x, j) \) being the order parameter in the \( j \)-th layer. The parameter \( \eta = m_{||}/m_{\perp} \) is the ratio of the effective mass parallel and perpendicular to the layers and \( s \) the interlayer spacing. Lengths are measured in units of the coherence length within the layers throughout the paper. We choose the \( y \)-axis perpendicular to the layers and the \( z \)-axis along the straight vortex line. Since we consider the extreme type 2 superconductors, we ignore magnetic field and merely require the order parameter changes its phase as we go around the vortex line. It is known that the vortex solution of Eq.(1) with lowest free energy has its center at a mid point between adjacent layers.\(^5\) We choose the origin of our \( xy \)-plane at the center so that the \( j \)-th layer is at \( y = (j - 1/2)s \).

Since we need a concrete form of the pair potential, we have solved the GL equation (1) numerically by the relaxation method. In the calculation we discretize the region \(-40 \leq x \leq 40\) of 20 layers \((j = -9 \sim 10)\) with mesh size 0.2. At the boundary of this region we require \( |\psi| = 1 \) and let the phase evolve so as to minimize the free energy. The relaxation step

\[
\psi^{(\text{new})}(x, j) = \psi(x, j) - c \frac{\partial F}{\partial \psi^*(x, j)} \quad (c : \text{constant})
\]

is repeated until the condition \( |\partial F/\partial \psi^*(x, j)| < 10^{-10} \) is satisfied for each \( \psi(x, j) \). Figure 1 shows the resulting order parameter in the case Josephson length \( s/\sqrt{\eta} = 2.5 \) appropriate to YBCO. Note that even in the two layers closest to the vortex center \((j = 0, 1)\) the amplitude is depressed only to 0.75 and that the size of the depressed region is of the order of Josephson length which is much larger than the coherence length. To see that this solution carries vortex current we have calculated

![Fig. 1. Amplitude (a) and phase (b) of pair potential for the \( j \)-th layer. \((j = 1, 2, \ldots, 5.\)](image-url)