We have earlier studied the diffusion of hydrogen in unstressed plate, where the hydrogen originated
from moving and stationary sources [1].

Let us examine the conjoint effect of external stresses and hydrogen diffusion on the propagation of
-cracks in a plate having a thickness equal to 2h.

It is known [2-4] that a plastic deformation zone is formed in front of a propagating crack. In this case
distribution of stresses at the crack front is shown in Fig. 1 [4], where Od represents the crack, point d
denotes the crack front, point d + a is the edge of the plastic zone opposite the crack front, K is the stress in-
tensity coefficient, and σ_T is the plastic flow stress. In view of its narrowness, length of the plastic deforma-
tion zone along the y axis may be considered as infinite.

The objective of the present work is to determine the concentration distribution, in the plastic region of
hydrogen, produced by a stationary source.

The effect of stress on the diffusion of hydrogen in a material can be found through selection of its chem-
ical potential as [5]

\[ \mu = \alpha C - \gamma \sigma, \]

where α and γ are constants and σ = σ_x + σ_y.

We can then write Fick's First and Second Laws in the following manner [6];

\[ I = -D \Phi C + M \Phi; \frac{\partial C}{\partial t} = D \Delta C, \]  \hspace{1cm} (1)

where D is the diffusion coefficient and M is a constant.

Stresses caused by hydrogen diffusion are neglected. We can arbitrarily divide stress distribution in a
plate into two regions: I. 0 < x ≤ d + a; II. d + a < x < ∞.

For short time intervals the diffusion of hydrogen in to the plate is described by the equations

\[ D_1 \left( \frac{\partial^2 C_1}{\partial x^2} + \frac{\partial^2 C_1}{\partial y^2} \right) \frac{\partial C_1}{\partial t} = \frac{\varepsilon_0}{2h} \delta(x - d) \delta(y) S_1(t), \quad 0 \leq x \leq d + a; \]

\[ D_2 \left( \frac{\partial^2 C_2}{\partial x^2} + \frac{\partial^2 C_2}{\partial y^2} \right) = \frac{\partial C_2}{\partial t}, \quad d + a < x < \infty, \]  \hspace{1cm} (2)

the solutions of which should satisfy the following initial and limiting conditions:

\[ C_1(x, y, 0) = 0, \quad \left. \frac{\partial C_1}{\partial x} \right|_{x=-m} = 0, \quad \mu_i(\infty, y, t) = 0; \quad i = 1,2 \]

\[ D_2 \frac{\partial C_2}{\partial x} \bigg|_{x=-m} = D_1 \frac{\partial C_1}{\partial x} \bigg|_{x=-m} + \frac{(1+\nu) M_1 K a}{3 \sqrt{2 \pi}} (y^2 + a^2)^{-\frac{\nu}{2}}; \]

\[ D_1 C_1(m, y, t) - M_1 \sigma = D_2 C_2(m, y, t) - \sqrt{\frac{2}{\pi}} \frac{(1+\nu) M_2 K a}{3} (y^2 + a^2)^{-\frac{\nu}{2}}. \]  \hspace{1cm} (3)
The latter two conditions correspond to continuous hydrogen diffusion and uninterrupted chemical potentials at the boundary of the two regions. $C_1$ and $C_2$ are the hydrogen concentration distributions in the first and second regions, respectively; and $m = d + a$. Stress distribution in the second region is given by the function

$$\sigma = \sqrt{\frac{2}{\pi}} \left(\frac{1}{3} \right)^{-\frac{1}{2}} \frac{K}{3} \left[ y^2 + (x - d)^2 \right]^{-1/4}.$$

Let us apply the Fourier and Laplace transforms, with respect to $y$ and time, respectively, to Eq. (2) and conditions (3). We then obtain, respectively:

$$\frac{d^2 \bar{C}_1}{dx^2} - \frac{\gamma_1^2}{12} \bar{C}_1 = -\frac{\epsilon_0}{2\sqrt{2\pi} hD_1 s} \delta(x - d), \quad \frac{d^2 \bar{C}_2}{dx^2} - \frac{\gamma_2^2}{12} \bar{C}_2 = 0;$$

$$\left. \frac{\partial \bar{C}_1}{\partial x} \right|_{x=m} = 0, \quad \left. \bar{C}_1 \right|_{x=m} = 0,$$

$$D_1 \frac{\partial \bar{C}_1}{\partial x} \bigg|_{x=-m} = \frac{4}{3} \frac{1 + \nu}{2\pi} \frac{dK_a}{\eta} \left( \frac{\eta}{2a} \right) \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)},$$

$$D_1 \frac{\partial \bar{C}_1}{\partial x} \bigg|_{x=-m} = \frac{4}{3} \frac{1 + \nu}{2\pi} \frac{dK_a}{\eta} \left( \frac{\eta}{2a} \right) \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)}.$$

where $\gamma_1^2 = \gamma_1^2 + s/D_1$; $\gamma_2^2 = \gamma_2^2 + s/D_2$.

The solution of Eq. (5) has the following form:

$$\bar{C}_1 = A_1 \cosh \gamma_1 x + B_1 \sinh \gamma_1 x + \frac{\epsilon_0}{4 \sqrt{2\pi} hD_1 s} \exp \left[ -\gamma_1 |x-d| \right], \quad \bar{C}_2 = A_2 e^{-\gamma_2 x} + B_2 e^{\gamma_2 x},$$

where $A_1$, $B_1$, $A_2$, and $B_2$ are integration constants determined from conditions (6). The final solution of Eq. (5) can be presented as follows:

$$\bar{C}_1 = \frac{\cosh \gamma_1 x}{D_1 (\gamma_1 \sinh m \gamma_1 + \gamma_1 \cosh m \gamma_1)} \left[ \frac{\epsilon_0}{8 \sqrt{2\pi} hs} \left( e^{-\sigma_1} + e^{\sigma_1} \right) \right.$$

$$+ e^{-|m+d| \gamma_1} - \frac{2i \gamma_1}{\gamma_1} e^{-\sigma_1} + \frac{i \gamma_1}{\gamma_1} e^{\sigma_1} - \frac{2i \gamma_1}{\gamma_1} e^{-|m+d| \gamma_1} \left] + \frac{2i a}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$

$$- \frac{\gamma_1}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$

$$+ \frac{\epsilon_0}{8 \sqrt{2\pi} hD_1 s} \left( e^{-|x+d| \gamma_1} - e^{-|x-d| \gamma_1} + 2 e^{-\gamma_1 |x-d|} \right);$$

$$\bar{C}_2 = \frac{\sinh m \gamma_1 \exp \left[ -\gamma_1 (x-m) \right]}{D_2 (\gamma_1 \sinh m \gamma_1 + \gamma_1 \cosh m \gamma_1)} \left[ \frac{\epsilon_0}{8 \sqrt{2\pi} hs} \left( e^{-\sigma_1} - e^{\sigma_1} \right) \right.$$

$$+ e^{-|m+d| \gamma_1} + \frac{2i \gamma_1}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$

$$- \frac{2 \gamma_1}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$

$$+ e^{-|x+d| \gamma_1} + \frac{2i \gamma_1}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$

$$+ e^{-|x-d| \gamma_1} + \frac{2i \gamma_1}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$

$$+ e^{-|x-d| \gamma_1} + \frac{2i \gamma_1}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$

$$+ e^{-|x+d| \gamma_1} + \frac{2i \gamma_1}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$

$$+ e^{-|x-d| \gamma_1} + \frac{2i \gamma_1}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$

$$+ e^{-|x+d| \gamma_1} + \frac{2i \gamma_1}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$

$$+ e^{-|x-d| \gamma_1} + \frac{2i \gamma_1}{s} \left( \frac{\eta}{2a} \right)^{-1/4} \bar{K}_{-\nu}^{(\eta)} \left( \eta \right) \right.$$