On the Phenomenology of Superconductivity in Cuprate Materials

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Consequences of d-wave pairing on the macroscopic properties of a superconductor are analyzed. A decisive experiment to probe the symmetry of the pairing wave function is discussed. It is shown that the magnetic properties of granular high temperature superconductors can be understood on the basis of d-wave superconductivity.

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Since more than a decade experimentalists and theorists are fascinated by the exotic superconductivity (SC) in heavy Fermion (HF) and CuO$_2$ materials which may be due to pairing with symmetry different from the conventional s-wave pairing. While for the former class of materials no unambiguous choice from a rather large number of possible SC states could be made yet, the controversy for the latter class is focused on conventional s-wave versus exotic d-wave pairing. For a long time experiments favored s-wave pairing. However, more recently several experimental findings seem rather compatible with d-wave pairing. Recent, NMR$^1$ and penetration depth measurements$^2$ have shown clear indication that there are low lying excitations contradicting the picture of a complete excitation gap. Angle resolved photoemission data even show that this low lying states are found along directions on the Fermi surface parallel to $k_x = \pm k_y^3$. This is compatible with a d-wave pairing state with the symmetry, $\psi(k) = k_x^2 - k_y^2$. All these experiments probe the density of states of the excitations in the superconducting state and not directly its symmetry. Therefore they could be also equally well be explained by assuming a very anisotropic s-wave pairing state with an extremely small, but finite gap along certain directions in the momentum space.

We will discuss here some phenomenological aspects which are specific to d-wave superconductivity. The usual starting point for this is the Ginzburg-Landau (GL) theory of SC. The GL functional of an order parameter with d-wave symmetry has a formal structure identical with that of an s-wave superconductor. At the superconducting phase transition no other symmetry is broken beside the U(1)-gauge symmetry. Therefore, it is not expected that there is any effect related with the phase transition and the critical fields which could distinguish between s- and d-wave symmetry. Practically all bulk phenomena found in conventional SC occur...
A more direct test of the symmetry of the order parameter must address the sign change of the pair wavefunction for different directions of \( \mathbf{k} \) on the Fermi surface. An ideal probe for this purpose is the Josephson effect, the phase coherent tunneling of Cooper pairs through an interface connecting two superconductors. A simple phenomenological description of the Josephson current phase relation in this case is given by

\[
J = J_0 f_1(n_1)f_2(n_2)\sin(\varphi_2 - \varphi_1)
\]

where \( J_0 \) is the maximal Josephson current.\(^4\) The orientation of the interface plays an important role, i.e.: the tunneling process is selective in momentum space. This is described by the functions \( f_i(n_i) \) which are symmetry functions of the interface normal vector \( n_i \) given in the basis of the crystal lattice of the superconductor \( i \). They have the angular dependence of the pair wave function which is completely symmetric in \( n \) for s-wave pairing, e.g.: \( f(n) = 1 \) but is anisotropic in the case of a d-wave superconductor, e.g.: \( f(n) = n_x^2 - n_y^2 \) or \( \cos(n_x) - \cos(n_y) \).

Depending on the orientations \( n_i \) of the interface on each side a phase shift \( \pi \) between an s- and a d-wave or between two d-wave superconductors can occur in addition to the phase difference \( \varphi_2 - \varphi_1 \). This phase can lead to frustration if we arrange the linked superconductors in a way that this phase shift cannot be simply removed by a U(1)-gauge transformation. For example, this is the case if we connect an s-wave superconductor at two surfaces perpendicular to the main axis of the \( x-y \)-plane of the d-wave superconductor (Fig.1). This loop has an intrinsic phase winding \( \pi \). Note that such a frustrating phase winding could never be generated in zero magnetic field by connecting two s-wave superconductors.

This intrinsic phase winding can be measured by an interference experiment in a specially designed SQUID as in Fig.1. A supercurrent \( I \) flowing through the SQUID in a way that the two junctions with the critical current \( I_1 \) and \( I_2 \), respectively, are working parallel, is subject to an interference effect which can be observed.

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Fig. 1. SQUID composed of an s- and d-wave superconductor.