Gravitational Radiation from a Binary System

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Abstract

It is shown that the gravitational radiation energy flux from a quasiperiodic binary system is given by the quadrupole formula contribution plus a nonlinear contribution which has been neglected in the past. The latter is found to depend on both the intensity of the gravitational field and the history of the radiating system. For PSR 1913+16, the nonlinear contribution to the energy flux may be neglected relative to that of the quadrupole formula.

Unlike Maxwell's equations, Einstein's field equations,

\[ R_{ik} - \frac{1}{2} R g_{ik} = \frac{8\pi G}{c^4} T_{ik} \]  

are nonlinear. For this reason, the problem of calculating the rate at which a system loses energy through gravitational radiation is more complicated than the corresponding electromagnetic wave problem. In 1918, Einstein [1] neglected nonlinear terms in (1) and showed that the approximate gravitational energy loss rate for a weak-field slowly moving system then satisfies the quadrupole formula

\[ - \frac{dE}{dt} = \frac{G}{45c^5} [\mathcal{D}^{\alpha\beta}]^2 \]  

Since this result is obtained from the linearized field equations, it is clearly valid for systems in which gravitational forces are negligible; the question that naturally arises is whether (2) is also valid for freely falling systems (in which gravitational forces are dominant), such as binary stars. This problem, which has been discussed throughout the history of general relativity,² is particularly urgent today in view

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² See, for example, citations in Refs. 3 and 4.
of recent observations [2] of the binary pulsar PSR 1913+16 which may establish
the existence of gravitational radiation. The purpose of this work is to clarify the
role of nonlinearities in the gravitational radiation mechanism for a weak-field
quasiperiodic binary system. In particular, conditions under which (2) is valid
are to be deduced.

The analysis below is an extension of the work of Cooperstock [3], who ini-
tiated a new approach to the general relativistic two-body problem. For a specific
free-fall collision, it was shown by an order of magnitude calculation that non-
linear contributions to the energy loss may invalidate the quadrupole formula.
In a further development [4], a model two-body weak-field slowly moving system
was analyzed in detail and, as anticipated in Ref. 3, the nonlinear contributions
to the energy flux were found to dominate those of the quadrupole formula.
Throughout these calculations, emphasis was laid on three important conditions
whose neglect has plagued many workers in the past [5]: the absence of incom-
ing radiation, a self-consistent development of solutions of (1), and the absence
of singularities in the metric. Thus, the quadrupole formula is invalid for at least
one type of physically realistic weak-field slow motion system. 3 The analysis be-
low begins with an examination of the usual derivation of the quadrupole formula
for the radiation energy loss with particular emphasis on nonlinear terms which
have been neglected in the past. An estimate is made of the magnitude of con-
tributions from nonlinearities outside the near zone; the contribution from other
neglected terms, which have recently been identified by Cooperstock and Hobill
[7], is evaluated exactly.

In the usual derivation [8, 9] of the quadrupole formula, a coordinate system
whose spatial origin lies in the vicinity of matter and which becomes asymptot-
ically Lorentzian at spatial infinity is employed. If the metric is defined
\[ g_{lm} = \eta_{lm} + h_{lm} \]  
(3)
where \( \eta_{lm} \) is the Minkowski form, then the imposition of coordinate conditions
\[ \psi_{,m} = 0; \quad \psi^{lm} = h^{lm} - \frac{1}{2} \eta^{lm} h_{kk} \]  
(4)
casts the exact field equations, (1), into the form (c = 1 and signature is +---)
\[ \Box \psi^{lm} = 16\pi G (T^{lm} + t^{lm}) \]  
(5)
where \( \Box \) is the flat-space wave operator and the \( t^{lm} \) are components of the
pseudotensor, which is a nonlinear function of the metric [9]. Equations (4)
and (5) yield
\[ (T^{lm} + t^{lm})_{,m} = 0 \]  
(6)

Although use of singularities leads to some ambiguities in the results, it is claimed that the
quadrupole formula fails also for this system.