Rotation in Cosmology†

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Cosmological models for a universe with expansion and rotation are considered. In particular, we analyse some effects of the universal rotation on the observational cosmology. It is shown that pure cosmic rotation does not produce neither causality violations, nor parallax effects, nor anisotropy of the microwave background radiation. It can be detected by studying angular dependence of standard cosmological tests, and is directly measurable via polarization observations. The latter are used to obtain experimental estimates for the direction and value of the rotation of the universe.

Noticing that most of physical objects (elementary particles, celestial bodies, stars, galaxies, etc) are rotating, one may wonder why the largest physical system — the universe — should be an exception. Here we shall not enter into a discussion of philosophical significance of cosmic rotation (though, in our opinion, the analysis of its relation to the Mach’s principle is of great interest). Instead, this paper will describe some effects on observational cosmology of the universal rotation.

Since the first studies of Lanczos, Gamow and Gödel [1,2], a great number of rotating cosmological models have been considered in the literature (these will not be reviewed here, see e.g. Refs. 3,4). Nevertheless the full understanding of observational manifestations of cosmic rotation is still far from reach. Moreover, there is a general belief that rotation of the universe is always a source of many undesirable consequences, most

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serious of which are timelike closed curves, parallax effects, and anisotropy of the microwave background radiation. The aim of this paper is twofold: to show that the above phenomena are not inevitable (and in fact, are not caused by rotation), and to find true effects of cosmic rotation.

Our starting points are two fundamental assumptions of Kristian and Sachs [5] which underlie the general analysis of observations in arbitrary cosmology: (i) the universe is described by a Riemannian space-time with sufficiently slowly varying metric, (ii) the light travels along null geodesics according to the geometric optics laws. In order to obtain certain numerical estimates of effects, Kristian and Sachs use also the third assumption that metric satisfies the Einstein gravitational field equations for dust. However the latter will be omitted here: for the sake of generality, as far as possible we shall not restrict ourselves to a particular dynamical model.

The spatial distribution of matter we observe at present is isotropic and homogeneous to a high degree. Therefore, it seems natural, so long as pure rotation effects are concerned, to consider a class of rotational shear-free spatially homogeneous models

$$ds^2 = dt^2 - 2R(t)n_i(x)dx^i dt - R^2(t)\gamma_{ij}(x)dx^idx^j.$$  (1)

Homogeneity here means the existence of a three-dimensional group of motions which acts simply transitively on $t = \text{const.}$ hypersurfaces; hence (1) describes Bianchi type space-times. These are classified according to the form of Killing vector fields $\xi^\mu_{(a)}$, $a = 1, 2, 3$, and their commutators $[\xi_{(a)}, \xi_{(b)}] = C_{ab}^{d}\xi_{(d)}$. Spatial metric components are then defined by

$$n_i = \mu_a e_i^{(a)}, \gamma_{ij} = \lambda_{ab} e_i^{(a)} e_j^{(b)}.$$  (2)

where $\mu_a, \lambda_{ab}$ are constant, and $e^{(a)} = e_i^{(a)} dx^i$ are relevant invariant one-forms, $L_{\xi_{(a)}} e^b = 0$. The explicit form of $C_{bc}^{a}, \xi_{(a)}, e^{(a)}$ is well known, see e.g. [4,6]. Let us choose the matrix $\lambda_{ab}$ to be positive definite which ensures that $t = \text{const.}$ slices are everywhere spacelike.

Space-times (1)-(2) are the simplest consistent cosmological models with rotation. Their kinematical properties are evident: the shear tensor is trivial $\sigma_{\mu\nu} = 0$, the volume expansion is $\theta = 3\dot{R}/R$, and vorticity is given by

$$\omega_{0i} = 0, \omega_{ij} = -\frac{R}{\dot{R}} \mu_a \left( \partial_i e_j^{(a)} - \partial_j e_i^{(a)} \right).$$  (3)

The value of rotation $\omega = (1/2 \omega_{\mu\nu} \omega^{\mu\nu})^{1/2}$ decreases in expanding universe, $\omega \sim 1/\dot{R}$. 