Solitonic Solutions on a Bianchi II Background Generated by SHEEP Algebraic Manipulation

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The exact solutions of Einstein's (vacuum) field equations corresponding to a (real-poles) three-solitonic perturbation of a Bianchi type II spacetime are investigated by using computer algebra. Two-dimensional and three-dimensional plots of the relevant solitonic fields are obtained numerically.

1. INTRODUCTION

Although the behavior-at-large of our universe seems to indicate that the universe is homogeneous and isotropic, it is commonly accepted that in the primordial stages of its evolution a stochastic background of gravitational waves was produced. These waves would generally have a longer period than the waves generated at the present epoch and they could be observable by new detection techniques [1,2]. It is therefore interesting to investigate exact solutions of Einstein's equations which represent perturbations over homogeneous (and possible isotropic) backgrounds, which in the far past behave as a sea of gravitational waves but evolve in the future towards the background itself. Inhomogeneous cosmological solutions admitting a

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two-parameter group $G_2$ of isometries provide a possible example of this situation, although, of course, they produce a very correlated gravitational wave background (all waves travelling in the same direction) rather than a stochastic background (like the one probably present in our universe). The soliton technique of Belinskii and Zakharov [3] gives a systematic way to generate this kind of behavior over vacuum and perfect fluid background metrics admitting a $G_2$. The technique allows us in principle to superpose an arbitrary number of waves onto the given background. However, the calculations become more and more complicated as this number grows, so that only solutions with few solitons have been considered so far. In [4] the one-solitonic solution on a Bianchi II background has been calculated and in [5] the solitonic fields related to this solution have been studied. In [6] the field of a two-solitonic solution on the same background has been discussed. Solitonic perturbations on other backgrounds have been studied by Verdaguer [7]. We have recently considered in detail the case of three interacting solitons by relying on algebraic computing and numerical methods to match all the difficulties. Although a solution with three solitons is rather far from describing a sea of primordial waves, it is the first case in which the interaction regime can be really seen at work. In this paper we aim at describing briefly the output for this case, by showing some of the diagrams which describe in two and three dimensions the behavior of a 3-solitonic perturbation over a Bianchi II background. The results were already announced in [8].

2. PRELIMINARIES

Let us recall that the Belinskii-Zakharov (hereafter BZ) inverse scattering technique applies to cosmological background solutions to produce exact perturbations which should correspond to the superposition of an arbitrary but finite number of solitary waves (see Refs. 3,4). The cosmological solutions looked for are locally expressed by

$$ds^2 = f(z,t)(-dt^2 + dz^2) + g_{ab}(t, z) dx^a dx^b,$$

where $a = 1, 2$ and $\|g_{ab}\|$ is positive definite. Einstein's (vacuum) equations split into two groups, one being non linear in $\|g_{ab}\|$ and the second one allowing us to calculate $f$ by quadratures once $\|g_{ab}\|$ has been found.

Having fixed the background metric

$$ds^2_0 = f_0(z, t)(-dt^2 + dz^2) + g_{ab}^0(t, z) dx^a dx^b,$$