Synchronized Frames for Gödel's Universe

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We exhibit Gödel's geometry in terms of a set of gaussian systems of coordinates, the union of which constitutes a complete cover for the whole manifold. We present a mechanism which induces a particle to follow a closed time-like line (CTL) present in this geometry. We generalize the construction of special class of observers (Generalized Milne Observers) which provides a way to define the largest causal domain allowing a standard field theory to be developed.

1. INTRODUCTION

A simple glance into any book of Relativistic Cosmology displays an interesting common characteristic: all cosmological models are depicted in gaussian systems of coordinates with just one remarkable exception, Gödel's 1949 rotating Universe [1].

This particularity is in general interpreted to be nothing but a consequence of the well-known impossibility of constructing a unique global gaussian system in this geometry. However, such a property does not forbid the use of a local gaussian system.

Indeed, the theory of Riemannian differentiable manifolds asserts that it is always possible, at least in a restricted domain, to represent point-events by means of a gaussian coordinate system. The extension of this system beyond a given domain depends on properties of the geometry at large.

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Although there have been some comments in the literature concerning synchronized systems of Gödel’s cosmological model, an explicit form has never appeared. We intend to remedy this situation in this article by exhibiting a set of gaussian systems of complementary domains, in such a way that their union constitutes a complete cover for the whole manifold.

The restriction on each synchronized frame can be understood as a consequence of the highly confining property of Gödel’s geometry. A question then arises: How can one reconcile such confinement with the homogeneity property of this metric? How could a point (any point) of such homogeneous space-time act as an irresistible attractor? This is precisely the condition to limit the extension of a chosen family of time-like geodesics, inhibiting it from going beyond a certain domain, and so restricting the region covered by the associated chart. To understand this one should look more carefully into the dynamical behaviour of free particle. Since the velocity of photons is the highest allowed one, let us just consider their propagation.

From electrodynamics and gravity standard coupling photons travel along null geodesics. Now, from the behaviour of geodesics in Gödel’s geometry [2,3] one obtains that the photons’ trajectory, which passes through a point $P$, can be equivalently described as if the particle feels an attraction to $P$ by a potential $V(r) = V_0 \tanh r$ (in which $V_0$ is a constant) having an energy $E < V_0$ [2]. This means that the net consequence of such a potential is to forbid the particle to leave the region $\mathcal{D}(P)$ which consists in the points encircling $P$ of a given radius. The actual value of the maximal allowable radius depends on the strength of the vorticity $\Omega$. Thus, any geodesic which passes an (arbitrary) point $P$ remains—for its complete history—confined in a cylinder around $P$ of radius $r_0$. This has an immediate consequence, which we referred to previously: if one displays a gaussian coordinate system from a point $O$ (arbitrary) then this system cannot be extended beyond $r_0$. This is a consequence of the dependence of the gaussian system on a particular choice of time-like geodesics $\Gamma(O)$ which precisely yields the identification of the local (gaussian) time to the proper time of $\Gamma(O)$.

We can build another gaussian system centered on another point $O'$ distinct from $O$. This new system can be located either within the domain of the previous Gauss-I system, in the region $0 < r < r_0$ or beyond it. We can then follow the same procedure as in the previous case and define a new gaussian chart (call it Gauss-II) based on point $O'$. This method can be repeated successively and complete the covering of the whole manifold. We present in Section 2 a short resumé of such peculiar behaviour of time-like geodesics in Gödel’s Universe.