Classical and Quantum Models of Strong Cosmic Censorship

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Abstract

The cosmic censorship conjecture states that naked singularities should not evolve from regular initial conditions in general relativity. In its strong form the conjecture asserts that space-times with Cauchy horizons must always be unstable and thus that the generic solution of Einstein’s equations must be inextendible beyond its maximal Cauchy development. In this paper we shall show that one can construct an infinite-dimensional family of extendible cosmological solutions similar to Taub-NUT space-time. However, we shall also show that each of these solutions is unstable in precisely the way demanded by strong cosmic censorship. Finally we show that quantum fluctuations in the metric always provide (though in an unexpectedly subtle way) the “generic perturbations” which destroy the Cauchy horizons in these models.

§(1): Introduction

One of the main outstanding problems in classical general relativity is to prove (or disprove) the cosmic censorship conjecture. This conjecture states (roughly) that naked singularities should not evolve from regular initial conditions in solutions of Einstein’s equations. The validity of this conjecture seems to be crucial for the ultimate viability of Einstein’s theory of gravitation. A naked singularity represents a breakdown in the classical picture of space-time structure and (presumably) signals the need for a quantum theory of gravitation. This would perhaps not be very surprising if the effects of such singularities were

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confined to the microscopic realm. But the most likely progenitor of a naked singularity, if such are indeed possible, is the catastrophic collapse of a massive stellar system. It would be most unsettling if the global observable consequences of such events could not be understood without a quantum theory of gravitation. In spite of its importance, however, there is little direct evidence for the correctness of the cosmic censorship conjecture.

There are two somewhat different forms of the cosmic censorship conjecture. The weak form refers only to asymptotically flat space-times and states that singularities which develop from regular initial conditions should always be hidden inside black holes. This weak conjecture is compatible with the occurrence of “locally naked” singularities such as those of the Reissner-Nordström solutions with $|Q| < M$.

The strong form of the cosmic censorship conjecture attempts to exclude even the locally naked singularities by claiming that they are exceptional. More precisely, the strong conjecture states that the generic, maximal Cauchy development of nonsingular initial data should be an inextendible (necessarily globally hyperbolic) space-time. This conjecture claims that the peculiar extendibility of, say, the Reissner-Nordström solutions is an unstable characteristic which the generic small perturbation would destroy.

Strong cosmic censorship makes sense even for cosmological space-times (i.e., space-times having compact spacelike hypersurfaces) and, if true, excludes (as nongeneric) other forms of pathological behavior such as that exhibited by Taub-NUT space-time. Taub space is a maximal globally hyperbolic space-time which is, however, extendible across a Cauchy horizon into NUT (Newman-Uni-Tamburino) space. NUT space has closed timelike lines through each of its points and represents an absurd breakdown of causality, even though, in a sense, it is otherwise nonsingular. Strong cosmic censorship asserts that Taub-NUT space is unstable and that a generic small perturbation would destroy its Cauchy horizon.

The stability properties of both Taub space and Reissner-Nordström space have been studied using linear perturbation theory and there is good evidence for the validity of strong cosmic censorship in these particular cases. But what of the general solution of Einstein’s equations? How many extendible solutions are there and are they all unstable? What is the role of quantum effects in providing the “generic perturbations” which presumably destroy the Cauchy horizons? In this essay we shall discuss these questions for the special case of Gowdy [2, 3] metrics on the manifold $T^3 \times R$. We shall show that this class of space-times contains an infinite-dimensional family of extendible solutions of Taub-NUT type but that each member of this family is indeed unstable in the sense required by strong cosmic censorship—a generic small perturbation destroys its Cauchy horizon.

For a recent review see Ref. 1.