Thermodynamics and General Relativity Could Determine the Geometry of the Universe

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We introduce a suggestive model where certain quantities in Friedmann models are treated like their thermodynamic counterparts: temperature entropy, Gibbs energy, and so on. Within this model, changes in the symmetry of the universe are interpreted as first- or second-order phase transitions. The thermodynamics we introduce give us a new way of determining the geometry of the universe. By choosing a specific local equation of state \( P = \alpha \rho \), we show that with respect to the thermodynamics we have introduced, it is always more advantageous for the universe to be in a Bianchi V (open) symmetric state.

1. INTRODUCTION

Behavior of black hole parameters (area, surface gravity, and so on) like certain thermodynamic quantities (entropy, temperature, and so on) motivated Bekenstein [1] to conjecture the existence of black hole thermodynamics. Later, the discovery of black hole radiation by Hawking established the physical link between these parameters and their thermodynamic counterparts [1]. However, despite the success of black hole thermodynamics, the relation between general relativity and thermodynamics remains to be established for more general metrics [2]. In this essay, in order to explore this relation, we consider the possibility of the Bianchi symmetry of a Friedmann model changing as the universe evolves (Bianchi I, IX, and V symmetric Friedmann models are also known as critically open, closed, and open Friedmann models.).

It is well-known that symmetry changes in thermodynamics can be
associated with phase transitions [3]. A typical example is the phase transition from \( \alpha \)-iron to \( \gamma \)-iron, where the symmetry of the lattice over which the iron atoms are distributed changes from \( bcc \) to \( fcc \) [3]. It would be interesting if the gas of galaxies, distributed over curved space, displayed an analogous phase transition when the Bianchi symmetry of the universe is allowed to change. Since the Bianchi symmetry refers to the symmetry of the three-dimensional space, Friedmann models are particularly useful in demonstrating the possible existence of such phase transitions.

Phase transitions are also due to the collective behavior of gas molecules. Hence they are governed by the entropy criteria rather than the energy. A well-known example of this fact is the existence of white tin in stable form at 298 K, a form which should be unstable at that temperature according to the energy criteria [3]. In order to demonstrate the existence of similar phase transitions among Friedmann models, we need to identify the corresponding quantities that play the roles of temperature, entropy, and so on. Establishing such a direct relation between arbitrary gravitational fields and thermodynamics cannot be accomplished solely by the use of equilibrium thermodynamics. However, Friedmann models (granted that they change with time sufficiently slowly) have certain properties like a universal (classical) time coordinate and uniform spatial curvature that allow us to use equilibrium thermodynamics. One of the characteristics of equilibrium thermodynamics is the existence of a temperature, which is invariant under spatial coordinate transformations (a three-scalar), and which is uniform throughout the system. The suggestive model we use is the one in which the radius of curvature of three-space (another three-scalar) is treated like the inverse of temperature, and \( \rho(P, T) \) plays the role of the Gibbs potential energy density. We show that for the transitions between Bianchi I and V, and Bianchi I and IX symmetric models, there is only one Gibbs function, and the transformation is of second-order. For the transformations between Bianchi V and Bianchi IX symmetric models we have two distinct Gibbs functions, and in general this leads us to first-order phase transitions. We also discuss a specific case which demonstrates an interesting prediction of the model that agrees with current observations.

2. THE PHASE TRANSITIONS

The field equations for the Friedmann models with Bianchi IX or I symmetry and for a perfect fluid are given as \([4]^4\)

\[
8\pi P = -(1/R^2)e^{-\xi} - \bar{g} - \frac{3}{4}\bar{g}^2
\]

\(^4\)Friedmann models are represented by the metric \( ds^2 = dt^2 - e^{\xi(t)}\{[dr^2/(1 - r^2/R^2)] + r^2d\Omega^2} \).