Abstract

It is shown that the effect of a gravitational field on a hydrogen atom is to admix states of opposite parity such as $2S_1/2$ and $2P_1/2$. The phase of this admixture is such as to produce circular polarization of the radiation emitted in transitions such as $2S_1/2 \rightarrow 1S_1/2 + \gamma$ which arises from the interference between the gravity-induced amplitude and that due to the weak neutral current. The predicted magnitude of the circular polarization, which could be sufficiently large to be detected in white dwarfs or in certain binary systems, varies from theory to theory. It is thus possible that a study of this effect could provide a feasible means of testing general relativity at the quantum level.

In a recent series of elegant experiments Colella, Overhauser, and Werner [1] (COW) have established that the quantum-mechanical behavior of thermal neutrons in a gravitational field is exactly as predicted by Newtonian gravity. Since all known theories of quantum gravity reduce to the Newtonian result in the nonrelativistic limit, their result, while important, does not allow a discrimination among competing theories at the quantum level (e.g., Einstein versus Brans-Dicke). To achieve the necessary discrimination COW would have to measure quantum effects that are characteristically smaller than the leading Newtonian contribution by a factor of $\beta^2 = (v/c)^2$, where $v$ is the velocity of the particles being studied and $c$ is the speed of light. Since $\beta \approx 10^{-5}$ in the COW experiment there appears to be little hope of refining this experiment to achieve the necessary sensitivity. Other proposed experiments, such as those aimed at detecting the spin precession of slow neutrons in a gravitational field,

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1 This essay received the fourth award from the Gravity Research Foundation for the year 1979—Ed.
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are also approximately 10 orders of magnitude below the required sensitivity. It is, however, extremely important to devise tests of relativistic quantum gravity (RQG) since there is always the possibility that new gravitational effects could manifest themselves at the quantum level whose existence would not have been anticipated by an extrapolation from the macroscopic realm.

As we will see shortly, the scale of RQG effects is set by the product of $\beta$ and a constant $\eta$,

$$\eta = \frac{g\hbar}{c}$$  \hspace{1cm} (1)

where $g$ is the local acceleration of gravity and $\hbar$ is Planck's constant. At the surface of the Earth $\eta = 2.15 \times 10^{-23}$ eV, while for a typical neutron star $\eta \approx 10^{-12}$ eV. Although there is nothing that can be done about the intrinsic weakness of the gravitational interaction, we can hope to find systems in which RQG effects are enhanced because both $\beta$ and $\eta$ are large, and which at the same time allow these effects to be studied by means of experimental techniques capable of great precision. What we wish to point out here for the first time is that both of these hopes may in fact be realized by the study of a new phenomenon, namely, the violation of parity selection rules for electromagnetic radiation induced by a gravitational field.

We begin with the generally covariant Dirac equation for an electron in a gravitational field, expressed in terms of a set of matrices $\gamma^\mu(x)$ which satisfy

$$\gamma^\mu(x)\gamma^\nu(x) + \gamma^\nu(x)\gamma^\mu(x) = 2g^{\mu\nu}(x)$$  \hspace{1cm} (2)

where $g^{\mu\nu}(x)$ is the metric tensor. The matrices $\gamma^\mu(x)$ are related to the usual (constant) Dirac matrices $\gamma^a (a = 1, 2, 3, 0)$ by

$$\gamma^\mu(x) = e^\mu_a(x)\gamma^a$$  \hspace{1cm} (3)

Here $e^\mu_a(x)$ are a set of tetrad fields which, for the case of a static spherically symmetric mass distribution, are given by

$$e^\mu_a(x) = \delta_{\mu a}(1 + \Phi) - 2\delta_{\mu 0}\delta_{a 0}\Phi$$

$$\Phi = \frac{-GM}{\rho c^2}$$  \hspace{1cm} (4)

Here $G$ is the Newtonian gravitational constant, $\rho$ is the distance from the electron to the center of the matter distribution whose mass is $M$, and we assume that we are in the weak field limit $|\Phi| \ll 1$. The Dirac equation obtained from equations (2)-(4) can be expressed in terms of an equivalent Schrödinger Hamiltonian by means of a Foldy–Wouthuysen transformation. Combining the resulting expression for the electron with the analogous result for a proton we find that the Hamiltonian for a hydrogen atom in an external gravitational field