A Spin-3/2 Theory of Gravitation

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Abstract

Stimulated by ideas occurring in supergravity, we develop a gauge theory of gravity based on a spin-3/2 Majorana field. Our theory has no metric or vierbein as an elementary field. Classically the theory is in complete agreement with Einstein's metric formulation, but quantum mechanically it differs from ordinary formulations, including supergravity, on the fundamental nature of gravitation. In our approach gravitation arises from a collective effect due to spin-3/2 gravitinos.

Supergravity [1–4] is a theory of gravitation and of a Majorana spin-3/2 field, invariant under a local supersymmetry transformation. Inspired by this idea we envisaged the possibility of constructing a theory of gravitation in which the metric tensor is obtained as a composite field in terms of the elementary spin-3/2 field alone [5].

In this essay we propose a generally covariant and locally Lorentz invariant theory of a Majorana spin-3/2 field $\psi^\alpha_\mu$. It differs from supergravity by the absence of an elementary vierbein field $h^\mu_\nu$. Nevertheless we show that through an appropriate ansatz, a classical equation can be obtained for a "collective" metric tensor, which exactly coincides with Einstein's equation in general relativity. Thus we interpret Einstein's theory of gravity as a special classical solution of our theory, similar in spirit to, e.g., soliton or monopole solutions of classical field equations. This classical solution provides a curved background on which the Fermi field is quantized by taking

$$\psi^\alpha_\mu = (\psi^\alpha_\mu)^{\text{classical}} + (\psi^\alpha_\mu)^{\text{quantum}}$$

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This approach is analogous to quantization in the presence of a background field, often used in general relativity.

In the language of the geometry of fibre bundles we consider a principal bundle with a four-dimensional space-time manifold $M_4$ as a base and the Lorentz group $G = SL(2C)$ as the structure group. A connection\(^3\) is introduced in terms of an equivariant horizontal form $\mathcal{A}$ with values in the Lie algebra of $G$, whose components (in a coordinate basis) are the gauge potentials $A^{ij}_\mu$. We also consider an associated bundle with the fibre $\psi$ being a real spinor-valued horizontal one-form belonging to the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ representation of $SL(2C)$. The induced connection is given by $A = \mathcal{A}^{ij}_\mu$, where $a_{ij} = \frac{1}{2} (\gamma_i \gamma_j - \gamma_j \gamma_i)$ are real Dirac matrices (Majorana representation), generators of Lorentz transformations. We define the covariant derivatives $D_\mu$ in a coordinate basis $\{\partial_\mu\}$ by

$$D_\mu = \partial_\mu + A^{ij}_\mu a_{ij} \tag{2}$$

The curvature $R$ is a horizontal two-form with values in the Lie algebra of $G$ whose components in a coordinate basis are given by

$$[D_\mu, D_\nu] = -R^{ij}_{\mu\nu} a_{ij} \tag{3}$$

or

$$R_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu + [A_\nu, A_\mu] \tag{4}$$

We take the action to be given by

$$S = \frac{1}{\kappa} \int (D\psi)^\alpha \wedge (D\psi)^\beta (\gamma_5)^{\alpha\beta} \tag{5}$$

where $\kappa$ is a constant, the pseudoscalar Dirac matrix $(\gamma_5)^{\alpha\beta}$ is real and antisymmetric, and we have used the notation $(D\psi)^\alpha = (D_\mu \psi_\nu)^\alpha \ dx^\mu \wedge dx^\nu$. The metric that raises and lowers spinor indices is the charge conjugation matrix $C = \gamma_0$ in the Majorana representation. Notice that the action (5) is nonvanishing only if the components of $\psi$ are noncommuting.

It is instructive at this point to compare this action with that for general relativity in the vierbein formalism. Introducing a vector-valued horizontal one-form $h$ (the vierbein) transforming as the $(\frac{1}{2}, 0)$ representation of $SL(2C)$, one can write Einstein's action in the form

$$S_{Einstein} = \frac{1}{\kappa} \int h^i \wedge R^{ijk} \wedge \epsilon^{ijkl} \tag{6}$$

On the other hand, upon integration by parts (5) becomes

$$S = \frac{1}{\kappa} \int \psi^\alpha \wedge R^{ijk} \wedge \psi^\beta a_{ij} \epsilon^{ijkl} \tag{7}$$

\(^3\)The connection is a vertical form. The horizontal form $\mathcal{A}$ is actually a connection difference that is the difference between the connection and a trivial connection corresponding to pure gauge (zero curvature).