On the Intrinsic Entropy of the Gravitational Field

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Abstract

I show that in linearized general relativity it is impossible to construct a detector by the use of which the quantum state of the linearized gravitational field could be reliably determined. This is because there is no material satisfying the positive energy condition which can serve as a good conductor or absorber of gravitational radiation over a finite range of frequencies. If this property is true of the full theory then one can conclude that a certain proportion of both the energy and information carried by a gravitational wave is irreversibly lost, and that there is a corresponding intrinsic entropy associated with any distribution of gravitational radiation.

§(1): Introduction

The suggestion that the gravitational field might serve as a universal source for disorder and irreversibility is perhaps the most intriguing result to have emerged from the investigations into the relationship between quantum and gravitational phenomena in recent years [1, 2]. This suggestion comes, first of all, from the work of Hawking [2] on black hole evaporation [3], where it is argued, quite plausibly, that in the process of formation and evaporation of a black hole information concerning the precise quantum state of the various fields in nature is necessarily lost. Thus, if the field is known initially, before the formation of the black hole, to be in a given pure state, the state following the evaporation of the black hole can only be specified probabilistically. The interaction with the gravitational field results in an increase of entropy of the field by
an amount approximately equal to the Beckenstein–Hawking entropy of the
black hole [3, 4]:

\[ S = 4\pi \left( \frac{M}{M_{\text{Planck}}} \right)^2 \]  

Further, in this situation quantum mechanics, by which is meant unitary deter-
ministic evolution of the quantum state, has broken down [2]. Moreover, as
there can be no evolution from a mixed state back to a pure state, or, equiva-
ently, as a result of the increase in the entropy of the field, we see that the
evolution of a black hole from its initial formation through evaporation is an
irreversible process.

In this paper I will show that in linearized general relativity an irreversible
degradation of energy and a corresponding loss of information occurs whenever
energy is converted from some other form into gravitational radiation. This is
because, once a certain amount of energy is transferred to the gravitational field
as a result of the production of gravitational radiation, it can never be fully re-
absorbed by matter. As I shall show, this follows from the fact that there exists
no stable material which is completely opaque to gravitational radiation. Fur-
thermore, it is impossible to construct a detector whose response to the state of
the gravitational field is efficient enough that it can be used to determine pre-
cisely the quantum state of the linearized gravitational field. A certain amount
of information concerning the state of the gravitational field cannot be recov-
ered by any possible measurements, and as a result, the best possible measure-
ments which could be made on the state of the gravitational field will result in
its description by means of a statistical ensemble rather than by a pure state.
Corresponding to this loss of energy and information to the gravitational field,
there must then be associated with gravitational radiation an intrinsic entropy
which is a measure of the irreversibility inherent in the process of conversion of
energy into the form of gravitational radiation.

This intrinsic entropy of gravitational radiation will be estimated, following
the demonstration of the claims made above, and will be seen to take a simple
form in terms of the average energy carried by the various modes of the linear-
ized gravitational field. It is in most circumstances not large, although we will see
that under optimal circumstances the rate of production can approach the rate
of entropy production by an evaporating black hole as the source approaches its
Schwarzschild radius. However, the existence of this intrinsic entropy in the lin-
earized theory may be extremely significant because if the same results are
found to apply to the full theory it will mean that to the extent that energy is
exchanged between matter and the gravitational field the description of physics
in terms of the unitary evolution of pure quantum states loses its operational
basis in terms of correspondence with experiments.