NOTE ON THE MOTION OF BLACK HOLES

M. DEMIAŃSKI and L.P. GRISHCHUK

Institute of Theoretical Physics, Warsaw University
Sternberg Astronomical Institute, Moscow

Received 4 February 1974

ABSTRACT

It is shown that from the point of view of a distant observer the gravitational field of a moving Schwarzschild black hole is the same as produced by an extended, non-rotating, spherical body of the same mass. According to him, a black hole follows the Newtonian equations of motion, though some quantities, for example the distance, lose their Newtonian meaning.

Recent observational results concerning the X-ray sources in double star systems [1] provide important evidence of the existence of black holes. Therefore black holes are interesting not only from the astrophysical point of view, but also as bodies moving according to the laws of celestial mechanics. By a black hole we mean an object collapsing towards its gravitational radius.

The problem in general relativity of the motion of self-gravitating bodies has been studied by many different methods [2-7]. Not all of these can be applied to determine the motion of black holes. For example, to obtain the equations of motion, Fock [5] uses a particular form of the energy momentum tensor, assuming from the very beginning that the dimensions of bodies are much larger than their gravitational radii. Obviously, a black hole does not satisfy this condition.

The method developed by Einstein, Infeld and Hoffmann [2] (the EIH method) is more suitable for studying the problem of the motion of black holes because it does not use any particular form of the energy-momentum tensor to describe sources. Gravitational field equations are solved in empty space and the sources are treated as singularities. To describe the motion of a singularity it is necessary to know some surface integrals over a 2-dimensional surface enclosing it. It is significant that they do not depend on the shape of the surface. The EIH method applies in the quasi-stationary approximation, namely, when the time variations of the field are much smaller than the spatial changes.
We will use the general philosophy of the EIH method to study the motion of black holes. Let us consider an isolated system consisting, for simplicity, of two bodies. Their distance apart, \( R \), we will choose to be much larger than their linear dimensions \( \lambda_1 \) and \( \lambda_2 \). The linear dimensions are not restricted to being much greater than the corresponding gravitational radii.

The space-time metric we will take in the form:

\[
g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},
\]

where \( \eta_{\mu\nu} \) describes the flat metric. \( h_{\mu\nu} \) can be expanded into a power series of some parameter:

\[
h_{\mu\nu} = \sum_{n=1}^{\infty} \varepsilon^n h_{\mu\nu}.
\]

The parameter \( \varepsilon \) has only a formal meaning and is used to collect terms of the proper order \([8]\). Let the orders of magnitude of derivatives of the metric be related by:

\[
\frac{\partial h_{\mu\nu}}{\partial \xi^0} \approx \varepsilon \frac{\partial h_{\mu\nu}}{\partial \xi^1}
\]

in the quasi-static approximation \( \varepsilon \ll 1 \).

From the physical point of view, a gravitational field produced by a system of moving particles is quasi-stationary if the relative velocities are small in comparison with the velocity of light.

One could look for a formal solution of the empty Einstein field equations in the form of equation (1) everywhere, except at some singular points. However, in order to get an acceptable solution incorporating only a finite number of terms of the formal expansion (2) it is necessary to consider only the regions in which (2) is convergent. Series appearing in the EIH method converge in the intermediate zone, namely, for the distances larger than the gravitational radii but smaller than the wavelengths of gravitational waves radiated by the system \([9]\). The condition \( R \gg \lambda_1, \lambda_2 \) assures the existence of the intermediate zone and consequently, the existence of a region of convergence.

To recover from the Einstein general theory of relativity the Newtonian theory of gravity in the limit \( c \to \infty \), one must take \( h_{00} = 0 = h_{0i} \) \([6]\). By a coordinate transformation which does not destroy the Cartesian coordinates at infinity and does not change the type of expansion (2), we could get rid of \( h_{00}, h_{0i}, h_{ij}, h_{ij} \) \([7]\). It

\[\dagger\] Greek indices run through 0,1,2,3, and Latin indices through 1,2,3.