ABSTRACT

It is shown that the manifestly covariant quantization of gravity correctly reproduces the classical Reissner-Nordström solution in the $\hbar \rightarrow 0$ limit. This is explicitly verified by evaluating the lowest order tree graph contribution to the vacuum expectation value of the gravitational field produced by a spherically symmetric c-number charged source. The generalization from a point source to that of finite extension is unavoidable if the 'trees' are not to lead to divergent expressions. Moreover, the mass which appears in the R-N solution is seen to be positive definite. For convenience, the source is taken to be a sphere of uniform charge and matter densities. Owing to a mass renormalization relating the total mass of the sphere to its bare mass, charge and invariant extension, both exterior and interior solutions may then be generated. This mass renormalization formula is in complete agreement with that obtained by purely classical reasoning.

§(1): INTRODUCTION

In deciding whether or not the conventional approaches to the quantization of gravity [1] will lead to a genuine description of quantum phenomena, it is of primary importance that such approaches...
first generate the classical solutions in the $\hbar \to 0$ limit. One must thus show that the tree graph contribution to the vacuum expectation value (henceforth referred to as VEV) of the quantum gravitational field in the presence of a classical source is a solution of the classical Einstein equations. Such a procedure must essentially correspond to an iterative solution of the Einstein equations, but proofs of how this correspondence comes about were only formal until M.J. Duff explicitly showed that this is indeed the case when one attempts to generate the Schwarzschild solution from the manifestly covariant theory of quantum gravity [3]. For this solution to be reproduced by a summation of tree diagrams it was necessary that a finitely extended $c$-number source be used and a distinction be drawn between the total mass of the source to its bare mass, i.e., the inertial mass that the source possesses in the limit of zero couplings.

In the same spirit, by looking at the tree graphs of quantum theory we shall explicitly evaluate the vacuum expectation value of the gravitational field in the presence of a charged source and show that the resulting solution is Reissner-Nordström [4] (R-N). For this physically more interesting situation (owing to the presence of both long-range fields), and considerably more complicated (since gravity couples to both the source and electromagnetic fields) an insight will be gained as to the nature of the R-N behaviour and the relation between total and bare masses with the charge.

The difficulties encountered with point charges have been discussed extensively within the framework of classical theory. Thus, Arnowitt, Deser and Misner [5] have shown that the invariant extension of charged particles in general relativity has a minimum below which no solution of the field equations exists, and with space-time developing intrinsic singularities for particle sizes less than this minimum. Their analysis was carried out by investigating spherical charged shells at moments of time symmetry. In a recent work [6] we have succeeded in verifying their results for continuous matter and charge distributions, and we have indicated the intimate connection of true space-time singularities to negative-valued matter densities.

In the perturbation series of quantum theory, the difficulties with point charges will manifest themselves even to the lowest order in Newton's constant $G$. Here the choice of a point source leads to divergent expressions for the tree graphs which have nothing in common with the R-N behaviour. Moreover, had the mass

\[ \frac{1}{\hbar} \frac{\partial}{\partial \phi} \left( \frac{\partial \phi}{\partial \phi} \right) \]

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