DETERMINATION OF STRESS CONCENTRATION IN A PLATE NEAR A HOLE IN A THREE-DIMENSIONAL TEST

S. D. Bobritskaya and A. L. Kvitka

Resolvents derived in [1, 2] on the basis of the variation-difference method for the deformation of solids of revolution with cyclic symmetrical effects have been used in the present work in a new approach to solving the Kirsch problem concerning the effect of circular holes on stress distribution in stretched plates.

The problem has been formulated in the following way. From an "infinite" plate with a hole of radius $a$, a circular plate with an outer ring of sufficiently large radius is chosen (Fig. 1).

It is assumed that the stress in the vicinity of a radius will be the same as in a plate without a hole.

At a given intensity $S$ of tensile stresses at the outer periphery of the selected circular plate, the following stress components are effective:

1. \[
\sigma_r = S \cos \theta = -\frac{1}{2} S \cos 2\theta;
\]

2. \[
\tau_{\phi r} = -\frac{1}{2} S \sin 2\theta.
\]

In the ordinary formulation, stress concentration is determined by solving the plane problem of elasticity theory [3].

We have made a more accurate solution of the given problem on the basis of equations of the three-dimensional elasticity theory. From this viewpoint we have examined a thick-walled cylinder bounded by lateral surfaces of radii $a$ and $b$ and having a height equal to the thickness of the plate.

Investigation of the stress-strain state of the selected thick-walled cylinder may be reduced to solution of the problem under the following effects: a) axisymmetric normal load of intensity $S/2$ uniformly distributed over the outer surface $r = b$ of the cylinder; b) cyclic symmetrical load, also applied to the outer surface of the cylinder, the component of which in the peripheral direction obeys the law

\[
\tau_{\phi} = \cos 2\theta;
\]

\[
\tau_{\phi z} = -\sin 2\theta.
\]

In both cases, stresses are absent on the end surfaces $z = -\delta/2$ of the investigated cylinder:

\[
\sigma_z = \tau_{rz} = \tau_{\phi z} = 0.
\]

On the meridional section of the cylinder we superimpose a grid with steps of $\Delta r = 6/8$ and $\Delta z = 6/12$. Axial symmetry of the cylinder and symmetry of the applied marginal load relative to the two planes permit us to introduce a quarter of the meridional section into the calculation (Fig. 2).

The resolvents and expressions for components of the stress tensor, and also for inhomogeneous solids of revolution of general form with asymmetrical effects, are shown in [1, 2]. Let us substitute in them the Lamé metric coefficients for cylindrical coordinates, setting the elastic characteristics of the material constant and making the load axisymmetric, and we obtain resolvent difference equations for a
A cylinder of constant rigidity in axisymmetric load S/2 (the first part of the problem). We show the first resolvent equation for node 9 (see Fig. 1):

$$-u_i k_i \frac{\Delta r}{2\Delta z} (1 - \nu) (r_i + r_9) (k_3 + k_5) - u_3 k_3 \frac{\Delta r}{2\Delta z} (1 - 2\nu) r_9 (k_1 + k_9) - u_9 k_9 \frac{\Delta r}{2\Delta z} (1 - \nu)$$

$$\times (r_i^2 + r_9) (k_3 + k_5) - u_i k_i \frac{\Delta r}{2\Delta z} (1 - 2\nu) r_9 (k_1 + k_9) + u_5 r_9 (k_3 + k_5)$$

$$\times \left( \frac{r_6 + r_8}{r_9} \right) + \frac{(1 - 2\nu)}{r_9} [k_1 (r_1 + r_9) + k_3 (r_3 + r_9)] + (k_1 + k_5) \frac{\Delta r}{2\Delta z} (1 - \nu) (r_1 + r_9)$$

$$\times (k_2 + k_7) + \frac{(1 - 2\nu)}{2\nu} \Delta r \Delta z (k_1 + k_5) (k_3 + k_5) + \frac{w_3 r_9 (k_2 - k_3)}{4} - \frac{w_3 k_3}{4}$$

$$+ \frac{w_9}{4} (k_4 - k_2 + k_6 - k_3) = \frac{1 - 2\nu}{4G} S \Delta z (k_3 + k_5) (k_2 - k_3) r_9,$$

where $u_i$ and $w_i$ ($i = 1, 2, \ldots, 9$) are displacements of the nodes in directions of the axes $r$ and $z$; $r_i$ ($i = 1, 5, 9$) represents radii of the nodes (see Fig. 1a); $k_i$ ($i = 1, 2, \ldots, 8$) is a coefficient of occurrence, bearing information concerning the position of a node relative to the circumference [1, 2]; $\nu$ is Poisson's ratio, and $G$ is the shear modulus of the material. The second resolvent equation, which we will not show here, has a similar form.

Since most interest lies in the distribution of the peripheral normal stress $\sigma_0$, let us write its difference expression

$$\sigma_0 = \left( u_1 \frac{1}{\Delta r} v \frac{r_1}{r_9} (k_2 + k_5) - u_5 \frac{k_3}{\Delta r} \frac{r_9}{r_5} (k_4 + k_8) + u_9 \frac{1}{\Delta r} \frac{1 - 2\nu}{1 - 2\nu} (k_4 - k_2)$$

$$+ k_3 - k_5 + \frac{\Delta r}{r_9} (k_2 + k_4 + k_6 + k_8) - w_3 \frac{k_3}{\Delta r} \frac{1 - 2\nu}{1 - 2\nu} (k_2 + k_8) + w_7 \frac{k_7}{\Delta r} \frac{1 - 2\nu}{1 - 2\nu}$$

$$\times (k_3 + k_5) + w_9 \frac{1}{\Delta r} \frac{1 - 2\nu}{1 - 2\nu} (k_2 - k_3) \right) \frac{2G}{(k_4 + k_8 + k_6 + k_2)}.$$

Fig. 1. Scheme of the disk, load, and layout of the grid.

Fig. 2. Stress distribution in the disk.