Nonlinear Electrodynamics in Space-times with Torsion

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A nonminimal and nongauge invariant description of nonlinear electrodynamics in space-times with torsion is given. It is shown that in the case of the Weitzenböck teleparallel space-time, massive photons are produced by a mechanism which involves the nonminimal coupling constant and the divergence of the torsion vector.

1. INTRODUCTION

It is well known that in order to preserve local gauge invariance \cite{1-6} a gauge field cannot be minimally coupled to the geometry of a space-time with torsion such as the Riemann-Cartan space $U_4$. Many attempts have been made to solve this problem. To mention just two, we may cite the attempt of de Sabbata and Gasperini \cite{1} to construct a "semiminimal" coupling principle for the electromagnetic field in a space with torsion and the construction of Hojman, Rosembaum and M. P. Ryan \cite{6}, identifying the usual concept of a gauge transformation and allowing torsion to propagate in a vacuum $U_4$. In this paper we adopt a different point of view. Instead of trying to preserve the local gauge invariance, we construct a nonminimal coupling between electromagnetism and gravitation.

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in a Riemann-Cartan space-time via a nonlinear electrodynamics coupled with torsion. Although a unification with other physical interactions like the weak and strong interactions [1] in $U_4$ is done, our approach provides a mechanism to generate massive photons [7] in a Riemann-Cartan space-time [3]. Nonlinear photons in the context of General Relativity were constructed some years ago by Novello and Salim [7]. In the next section we review the Proca field [8] equation in a Minkowski space $M_4$, and the nonlinear photons in a Riemannian space-time. In Section 3 we present a Lagrangian density of the "Einstein-Cartan-Proca" nonlinear electrodynamics in space-times with torsion and derive the Proca field equation in $U_4$ and the Maxwell's nonlinear field equation with source. Taking the linear approximation of the torsion fields and the Weitzenböck $T_4$ teleparallel space, where the only contribution to curvature comes from the torsion, we show that the photon mass is proportional to the divergence of the torsion field in the static $T_4$ space-time.

### 2. MASSIVE PHOTONS IN GENERAL RELATIVITY

Some experiments for the detection of the massive photons in the context of special relativity and even GR have been proposed by Goldhaber and Nieto [10]. On the theoretical framework a nonlinear electrodynamics in (pseudo-)Riemannian space-times has been proposed by Novello and Salim [7]. Let us review the Proca field equation $M_4$

$$\Box A^i + \left(\frac{\mu^2}{4\pi}\right) A^i = J^i$$

where $\Box = \partial_i \partial^i$ ($\partial_i \equiv \partial/\partial x^i$) is the D'Lambertian operator in $M_4$, $A^i (i = 0, 1, 2, 3)$ is the 4-vector electromagnetic potential and $J^i$ is the charge 4 current. In the case of a static charge placed at origin eq. (1) has the following solution [8]

$$A^0 = q(e^{-\mu r/r})$$

in spherical coordinates. Here $\mu$ represents the mass of the photon; eq. (2) implies a short range type interaction for this type of electrodynamics. Novello and Salim have obtained the following generalization of Maxwell's equations in $V_4$

$$\nabla_i F^{ij} = J^i - (\lambda/k) R(\{\}) A^i$$

where $R(\{\})$ is the Ricci scalar of GR, $k = (8\pi G/c^4)$ is the Einstein coupling constant, and $F^{ij}$ is the electromagnetic field tensor given by

$$F_{ij} = \partial_i A_j - \partial_j A_i.$$