The Parallel Transport of a Vector: Its Physical Meaning in Three Geometrical Unified Field Theories

J. A. Pullin

Received September 25, 1985

The physical interpretation of the parallel transport of a vector is analyzed in three classical geometric unified field theories: the theory of physical space and the theories of Weyl and Infeld. In physical space it is found to be a Fermi-Walker transport in the presence of electromagnetic interactions.

1. INTRODUCTION

In classical general relativity the parallel transport of a vector has a straightforward geometric interpretation. Geodesics parallel-transport their tangent vectors, and, therefore, from an initial vector \( \mathbf{V} \) at point \( A \) this operation creates at point \( B \) a new vector \( \mathbf{V}' \) such that if one links \( A \) to \( B \) by means of a geodesic, its tangent vector at \( A \) forms the same angle with \( \mathbf{V} \) that the tangent vector at \( B \) forms with \( \mathbf{V}' \).

Since in general relativity geodesics are world lines of free-falling test particles, it follows that the basis vectors of the "natural" reference system (that is, one free-falling with the particle) undergo parallel transport. Any vector parallel-transported will then have a fixed orientation with respect to this reference system under this operation.

The situation is not always so clear in geometric unified field theories where the space-time has a richer structure than a Riemannian one. In many of these theories, test particles no longer fall along geodesics and so the interpretation given for general relativity breaks down.

---

1 Instituto Balseiro, Universidad Nacional de Cuyo and Comisión Nacional de Energía Atómica, 8400 Bariloche, Río Negro, Argentina.
In this work we investigate the meaning of parallel transport in the theories of physical space, based upon a Friedman–Schouten geometry, Weyl’s theory with two curvatures, and the theory of Infeld based upon a space with torsion.

2. PHYSICAL SPACE

The unified theory of physical space (1) geometrizes electromagnetism and gravity in a four-dimensional Friedman–Schouten space-time. It has an asymmetric affine connection with a particular torsion tensor with four independent components as indicated by

\[ \Gamma_{[\mu\nu]}^\lambda = S_{\mu\nu}^\lambda = S_{[\lambda} \delta_{\nu]}^\lambda \]  

The affine connection differs from the Christoffel symbols

\[ \Gamma_{\mu
u}^\lambda = \left\{ \begin{array}{c} \lambda \\ \mu \\ \nu \end{array} \right\} + S_{\nu}^{\lambda} g_{\mu
u} - \delta_{\mu}^{\lambda} S_{\nu} \]  

and the space is metric

\[ \nabla_{\lambda} g^{\mu\nu} = 0 \]  

The field equations in the particle limit are as follows

\[ R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = - b(\sigma_0 u^\nu u_{\nu} + i \beta F^{\mu\nu}) \]  

\[ S^\mu = \frac{\sigma_0}{\rho_0} \left( \frac{1}{c^2} F^{\mu\nu} + i \beta g^{\mu\nu} \right) u_{\nu} \]  

where \( \beta \) is a constant to be determined, \( \rho_0 \) is the proper mass density of the particle, \( \sigma_0 \) is the proper charge density of the particle. These are the equations of the theory in the point-particle limit. We should note that \( R^{\mu\nu} \) is not a symmetric tensor any more since it is calculated from an asymmetric affine connection.

If we start with a parallel transport

\[ u^\nu \nabla_{\nu} V^\mu = 0 \]  

where the covariant derivative \( \nabla_{\nu} \) is taken with the full asymmetric affine connection in which we replace the affine connection by its definition (2.2). If into this definition we put the value for \( S_{\nu} \) given by the field equations (2.5), we obtain

\[ u^\nu \nabla_{\nu} V^\mu + \left( u^\nu \frac{\sigma_0}{\rho_0} F_{\nu}^{\mu} - u^\mu \frac{\sigma_0}{\rho_0} F_{\nu}^{\nu} \right) V_{\nu} = 0 \]