Gravitational Scattering of Electromagnetic Radiation

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Abstract

The scattering of electromagnetic radiation by linearized gravitational fields is studied to second order in a perturbation expansion. The incoming electromagnetic radiation can be of arbitrary multipole structure, and the gravitational fields are also taken to be advanced fields of arbitrary multipole structure. All electromagnetic multipole radiation is found to be scattered by gravitational monopole and time-varying dipole fields. No case has been found, however, in which any electromagnetic multipole radiation is scattered by gravitational fields of quadrupole or higher-order multipole structure. This lack of scattering is established for infinite classes of special cases, and is conjectured to hold in general. The results of the scattering analysis are applied to the case of electromagnetic radiation scattered by a moving mass. It is shown how the mass and velocity may be determined by a knowledge of the incident and scattered radiation.

§(1): Introduction

The question that we examine here is the following: Can the structure of a gravitational source be determined by "looking" at it with electromagnetic radiation? Specifically, we use a perturbation-theoretic approach to study the scattering of incoming (first-order) electromagnetic multipole radiation in the

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background space-time of a (first-order) gravitational multipole field. The incoming electromagnetic radiation can be of arbitrary multipole structure. The gravitational fields that we consider here are of arbitrary advanced (for reasons of mathematical convenience) multipole structure. The outgoing (second-order) scattered electromagnetic radiation is then examined to see to what extent the gravitational multipole moments are determined by a knowledge of the multipole structures of the incident (first-order) and scattered (second-order) electromagnetic fields.

The scattering of electromagnetic radiation by gravitational fields has, of course, been widely studied. Couch and Hallidy [1] have examined the interaction of retarded first-order electromagnetic dipole and quadrupole radiation with retarded first-order gravitational monopole and quadrupole fields. Their second-order electromagnetic fields show scattering by the gravitational monopole field, but not by the quadrupole field. These results are in agreement with those of the present paper. A number of other investigators have studied the scattering of electromagnetic radiation in the Schwarzschild or Kerr backgrounds (see, for example, [2-9]).

The main mathematical techniques used in this paper are reviewed in Section 2. The gravitational and electromagnetic fields are expressed in the Newman-Penrose formalism [10], and perturbation expansions are made of each field. A time-reversal technique [11] is also employed. These techniques are used in Section 3 to analyze the problem of the scattering of electromagnetic radiation by gravitational multipole fields of arbitrary order. The next three sections apply this analysis to the cases of monopole, dipole, and higher-order multipole backgrounds, respectively. Section 7 contains some discussion of the results, and as an example shows how the mass and velocity of a uniformly moving mass can be determined from a knowledge of the incident and scattered electromagnetic radiation.

§(2): The Formalism

We use the Newman-Penrose formalism, including the standard coordinate and tetrad conditions [10]. We assume the reader's familiarity with this formalism for the Einstein-Maxwell equations, as well as with the differential operator $\partial$ and spin-weighted spherical harmonics [12]. All quantities are expressed in terms of a perturbation expansion; e.g., the spin coefficient $\rho$ may be written as

$$\rho = \rho^0 + \rho^1 + \rho^2 + \ldots$$

where the superscript refers to the perturbative order. The zero-order terms are those of flat space-time, the first-order terms arise from the linearized theory, etc.

The physical problem that we study is indicated in the Penrose picture [13] of Figure 1. At advanced times, $v$, earlier than $v_1$ the electromagnetic field is zero. Then first-order electromagnetic radiation is incident from past null infinity, $\mathcal{I}^-$, between advanced times $v_1$ and $v_2$. We solve the second-order equations