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Generic and Nongeneric World Models

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Catastrophe theory methods are employed to obtain a new classification of those world models which can be presented in the form of gradient dynamical systems. Generic sets and structural stability of models in the potential space are strictly defined. It is shown that if a cosmological model is required to be Friedman and generic, it must be flat.

1. INTRODUCTION

Modern research in theoretical cosmology quite obviously shifts from founding and analyzing individual solutions to Einstein's field equations to investigating a space of all possible solutions and discovering how certain properties (such as, for example, existence of singularities or horizons, tendency to isotropization, appearance of carbon necessary "to make a physicist...") are distributed in this space. A property is believed to be "physically realistic" if it can be attributed to "large" subsets of models within a space of all possible solutions or if it possesses a certain stability, i.e., if it is shared by a "slightly perturbed" model. Moreover, starting from Misner's chaotic cosmology program [1] many cosmologists believe that the actually observed Universe must have evolved from "average" initial conditions. These tendencies in contemporary cosmology inspired G. F. R. Ellis to formulate what is called by him a *probability principle*, which states that, "The universe model should be one that is a probable model within

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the set of all universe models," and a *stability assumption*, closely related to the above principle, "stating that the universe should be stable to perturbations" [2, p. 50].

Needless to say, these pronouncements have no meaning at all unless a probabilistic measure, or at least a suitable topology, is defined on the set of all possible solutions, which is not an easy task. After some research work having been done in this field (see, e.g., [3–5]), only partial knowledge has emerged concerning the structure of certain regions of the all-solutions space. For example, it has turned out that a subspace of all spatially closed space-times, which are solutions to the sourceless Einstein field equations, has a cone-like singularity at all its points representing space-times with symmetries. In other words, solutions in a neighborhood of such space-times cannot be smoothly parametrized by elements of a linear space. Therefore, it may easily occur that a perturbation of a symmetric solution might produce a space-time which in no reasonable sense could be regarded as an approximation to the original solution.

The problem of how to define "large" or generic sets within the space of all solutions, and consequently of how to understand structural stability, can be dealt with in a strict manner when it is restricted to a subspace of those world models that can be presented in the form of *gradient* dynamical systems. This is the subject matter of the present study. Mathematical tools elaborated by the so-called catastrophe theory provide us with the necessary means to cope with the problem. As a by-product one obtains a new classification of this subclass of models. Having at our disposal the gradient function, we are able to draw conclusions from the Thom–Mather theorem [6], together with the Morse lemma [7], concerning the generic or nongeneric character of certain model subclasses within the space of potentials. Then the structural stability problem becomes a mere consequence of the obtained results.

We treat catastrophe theory in an instrumental way. Discussion about the nature of the catastrophe theory and its philosophy (see, for instance, [8, 9]) is irrelevant as far as our aims are concerned; our use of the theory turned out to be only sound mathematics.

In Section 2 we briefly review those relativistic world models which can be put into the form of gradient dynamical systems. Section 3 gives some fundamental notions of the catastrophe theory which will be used for subsequent considerations. Once the potential function is given, the "catastrophic" classification of solutions is a routine matter; the results, mainly in the form of tables, are shown in Section 3. More about the "potential space of solutions" can be found in Section 4; generic and nongeneric world models and their structural stability properties are also discussed here.