EVALUATING THE LOAD-CARRYING CAPACITY OF THIN-WALLED TUBES
FROM THE INITIAL MOMENT OF STRAIN LOCALIZATION

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Based on the criterion of an unlimited increase in strain rate at the moment of fracture [1], investigators [2] have derived relations for determining the limiting load-carrying capacity of thin-walled tubes loaded with internal pressure and an axial force under creep conditions and have compared their results with empirical data.

In the present study, we use the earlier-derived formulas to predict the initial moment of strain localization in a tube of plastically deformed material not experiencing creep.

Since time was not included in the formulas [2] derived earlier to determine the limiting strains of thin-walled tubes in creep, this allows them to be used in cases where time is not a factor in the relationship between stress and strain.

As is known, in plasticity theory such a problem is solved by criteria of strain stability derived by different methods [3-7] and based as a rule on flow-theory equations of the von Mises type. These criteria are most often described in the form of the dependence of the critical subtangent $z_\ast$ [3-6] to the strain-hardening curve on $\xi = \sigma_Y/\sigma_X$ (where $\sigma_X$ and $\sigma_Y$ are the axial and peripheral true stresses) in the coordinates stress intensity $\sigma_e$--strain rate $\varepsilon_e$. The transition to the above variables from those used in [2] -- $\xi_0 = \sigma_Y/\sigma_{X_0}$ ($\sigma_{X_0}$ and $\sigma_Y$ are the nominal stresses, proportional to the loads) and $\rho$ (ratio of radii of thin-walled tube in the deformed and initial states) -- may be accomplished by means of the relations

$$
\begin{align}
\xi_0 &= \frac{2\sigma}{2\sigma + \rho^2 + \rho^2 + \rho^2} \quad ; \\
z &= \frac{2m}{2k - 1} \ln \rho.
\end{align}
$$

We should note that Eq. (1) was derived from Eq. (3) in [2], while Eq. (2) was derived using the condition of similitude of the stress and strain deviators together with certain values of $z$ and $\varepsilon_e$ with a strain-hardening law $\varepsilon_e = B\sigma^m$, where $B = B(\xi)$ and $m = \text{const}$.

Substituting Eqs. (1) and (2) in Eqs. (11) and (12) (applicable to the case of constant loads) in [2], we obtain the following equation for $z_\ast$:

Here, as in [2], the asterisk denotes a limiting value.

Equation (3) may also be derived from Eq. (7) in [6] if we determine the angle $\varphi$ between the stress vector and strain increment vector from the classical condition of the existence under steady loads of two infinitely similar stress states, determined by the condition of similitude of the stress and strain deviators contained in the starting equation for $\varepsilon_e = B\sigma^m$ (Eq. (1) in [2]). In this connection, it becomes obvious that under the conditions given the critical state will be determined only by the current values of the stresses and will be independent of the strain history.

Here

\[
C(\kappa) = (m - 1)(2 - \kappa)(2\kappa - 1) \frac{\kappa}{A} \frac{\partial A}{\partial \kappa} + 3\kappa + m(2 - \kappa),
\]

where, in accordance with Eq. (1) here and Eq. (4) in [2], the value of A is determined from the formula

\[
A = \frac{\sigma_q}{\sigma_\kappa} \cdot \frac{2 - \kappa}{2 - \kappa},
\]

(\sigma_q is the equivalent stress \([1, 2]\), making it possible to show the divergence of the strain curves in the coordinates \(\sigma_\kappa - \varepsilon_\kappa\)).

It is known (see [8, 9], for example) that the strain curves of many isotropic metals diverge in the coordinates \(\sigma_\kappa - \varepsilon_\kappa\) such that better agreement with the empirical data is often obtained by plotting a single curve in the coordinates maximum shear stress \(\tau_\kappa\)-maximum displacement \(\gamma_\kappa\). In this regard, it is best if we define more specifically Eqs. (4) and (5) both for the case when A is determined by the condition of the invariant function \(\sigma_\kappa - \varepsilon_\kappa\) and, in accordance with (5), has the form

\[
A = \frac{2 - \kappa}{2 - \kappa} \sqrt{1 - \kappa + \kappa^2},
\]

and for the case of a single curve \(\tau_\kappa - \gamma_\kappa\). In the latter case, it is necessary to examine two variants of the ratio between tensile stresses (\(\sigma_\kappa > \sigma_\gamma\) and \(\sigma_\gamma > \sigma_\kappa\)), which accordingly yields

\[
A = \frac{2 - \kappa}{2 - \kappa} \kappa \quad (\kappa < 1);
\]

\[
A = \frac{2 - \kappa}{2 - \kappa} \kappa \quad (\kappa > 1).
\]

Substituting (6) and (7) in (4), we have, respectively,

\[
C(\kappa) = 3\kappa + m(2 - \kappa) + \frac{3\kappa^2(m - 1)(2\kappa - 1)}{2(1 - \kappa + \kappa^2)} \quad \text{at} \quad \kappa < 1;
\]

\[
C(\kappa) = 2[m(1 - \kappa + \kappa^2) + \kappa(2 - \kappa)] \quad \text{at} \quad \kappa > 1.
\]

Let us determine the dependence of the limiting curves \(z_* = z_*(\kappa)\) on \(m\) in the plane of stresses (see Fig. 1). As follows from Eq. (4), at \(m = 1\) the value of \(C(\kappa)\) is independent of the type of generalized strain curve and the solution to Eq. (3) has a simple form† (it corresponds to solid lines AB and BC in Fig. 1):

\[
z_* = \frac{2\sqrt{1 - \kappa + \kappa^2}}{2 - \kappa} \quad \text{at} \quad \kappa \leq 0.5;
\]

\[
z_* = \frac{2\sqrt{1 - \kappa + \kappa^2}}{3\kappa} \quad \text{at} \quad \kappa \geq 0.5.
\]

Criteria (11) and (12) coincide with the conditions of Lankford–Saibel [3–7] obtained from the maximum of one of the loads.

With the other extreme value of parameter \(m\) (range of variation \(1 \leq m \leq \omega\), Eqs. (3) and (8) yield Swift's criterion [4, 5] (dashed line in Fig. 1):

\[
z_* = \frac{4(1 - \kappa + \kappa^2)\sqrt{2}}{4 - 6\kappa + 3\kappa^2 + 4\kappa^4}.
\]

After substituting Eq. (10) in (3), we obtain criterion (12), which proves to be valid for all \(m\). From Eqs. (3) and (9), as \(m \to \omega\) we have a criterion which is not in the literature (dashed line in Fig. 1):†

†As can be seen from Fig. 1, the boundary between criteria (11) and (12) goes along the ray \(\kappa = 0.5\), a fact due to the unsymmetrical nature of the tube's deformation relative to the stress state \(\kappa = 1\).