APPLICATION OF THE STATISTICAL-MODELING METHOD
FOR PREDICTING THE CYCLIC STRENGTH OF MATERIALS
UNDER RANDOM LOADING

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The method of statistical modeling is used to study the laws governing the fatigue failure of design com-
ponents and specimens subjected to random loading [1-5]. Making use of this method, a statistical loading
model is selected and a segment of the random process of stress variation in the high-risk section is modeled
on an electronic computer. The random process is then schematized, and represented in the form of a sequence
of cycles with a known amplitude and average value. The longevity of the component (specimen) is subsequently
computed on the basis of a fatigue-damage-accumulation model.

Itagaki and Shinozuka [1] and Bolotin et al. [2] have proposed that the random loading process is narrow-
band and the extremum of this process be modeled as a succession of correlated random numbers with a Ray-
leigh distribution. Wirshing and Shehata [3] and Kikukawa et al. [4] have employed the Gaussian random pro-
cess (GRP) with a given spectral density, modeled by summing the harmonics with random phases [6], as loading
models. Kobtsev and Perepelka [5] propose to realize the random process by modeling the oscillations of
a system with several degrees of freedom under a Gaussian white noise with allowance for nonlinear energy
dissipation. In these studies, the theory of linear damage summation and various methods of schematizing
random loading were used to estimate fatigue damage. The possibility of using any damage-accumulation mo-
del and method for schematizing the loading process for various loading models (e.g., a random Gaussian pro-
cess with a given spectrum, summation of random and polyharmonic processes, etc.) is an advantage of the
statistical-modeling method.

This report describes the method for modeling the segment of GRP realization for a given spectral den-
sity. The method is used to model the process of random-load variation during fatigue testing. Corresponding
spectral densities and the results of tests for random and sinusoidal loadings of alloy 2024-T3 (analog DI6) are
taken from [7-9]. The loading process was schematized by the full-scale method [10] with allowance for the
average value of the cycle.

Computer Modeling of the Random Gaussian Process. The following modification of the familiar expansion
of the random process in a Fourier series was used to model the GRP for a given spectral density \( S(f) \) (\( f \) is frequency) [6]. The ordinates \( x_m \) of the process, which are equidistant in time by \( \Delta t \), were computed from the equation

\[
x_m = \frac{1}{N_p} \sum_{k=0}^{N_p-1} V_k A_k \exp \left( i \frac{2\pi m k}{N_p} \right), \quad m = 0, 1, \ldots, N_p - 1,
\]

(1)

where \( N_p \) is the number of points in the realization being modeled, \( i^2 = -1 \), and \( v_k \) is a complex Gaussian ran-
dom value, in which case

\[
V_k = V_{N_p - k}^*, \quad \langle V_k V_k^* \rangle = N_p \delta_{kk}
\]

(2)

(3)

(where \( V^* \) is the complex conjugate of \( V \), \( \langle \cdot \rangle \) is an averaging symbol, \( \delta_{kk} = 1 \), and \( \delta_{ik} = 0 \) when \( i \neq k \)). The coefficients \( A_k \) in Eq. (1) are computed in the following manner:

Equation (6) relates the frequency interval $\Delta f$, the time interval $\Delta t$, and the number of reading points $N_p$. Making use of (2)-(5), we can easily prove this equality

$$R(n\Delta t) = \langle x_m x_{m+n} \rangle = \frac{1}{N_p} \sum_{k=0}^{N_p-1} S(\Delta fk) \exp \left( i \frac{2\pi n}{N_p} \right).$$

for the correlation function of the random quantity $x_m$. It follows from Eqs. (1) and (7) that blocks of numbers for $x_m$ and $R(n\Delta t)$ can be obtained, respectively, from $V_k A_k$ and $S(\Delta fk)$ blocks using the Froude inverse discrete transformation (FIDT) [11]. It is established on the basis of FIDT properties with consideration of equalities (2) and (5) that $x_m$ and $R(n\Delta t)$ are real numbers. The above makes it possible to consider that for a rather large $N_p$, the block of numbers $x_m$ assumes the value of the standard GRP with a zero average value and spectral density $S(0)/N_p$. The rms value of the process

$$\sigma_x = \sqrt{\langle x_m^2 \rangle} = \sqrt{R(0)} = \left( \frac{1}{N_p} \sum_{k=0}^{N_p-1} A_k^2 \right)^{1/2}.$$

It is apparent from (1), (4), and (6) that in reproducing the spectrum of the process

$$\Delta f \left( \frac{N_p}{2} - 1 \right) \approx \frac{\Delta f \cdot N_p}{2} = \frac{\Delta f}{2\Delta t}.$$

This means that the number of points modeled in the harmonic period with a maximum amplitude $\Delta f N_p/2$ is two. The extrema of the process must be determined to hold the number of points in the period under eight. If the spectral density of the process $S(f)$ is assigned in the interval $0 \leq f \leq f_m$, therefore, $\Delta f$ and $\Delta t$ should be

$$\Delta f = \frac{f_m}{N_p}; \quad \Delta t = \frac{1}{2f_m}.$$