Many structural elements in modern technology are subjected during operation to severe nonsteady thermal effects which cause high surface thermal stresses. In the case of cyclic heat exchange these stresses may lead to surface layer cracking even in quite ductile materials.

In studying the nonsteady thermal stress state of surface layers it is desirable to use approximate design schemes. For example, the wall of a cooled turbine blade may be substituted by a flat plate. This design scheme is suitable for many structural elements. It is assumed that heat transfer with nonsteady schedules proceeds only normal to the plate surface. This assumption, used many times previously [1, 2, and others], does not lead to marked errors.

The adopted design scheme of a plate with a heat flux normal to the surface may also be used for evaluating stresses and strains in surface layers of coated parts.

**Basic Equations.** The starting point of the coordinate system $xyz$ (Fig. 1) is located at the center of the plate on the initial surface situated at distances $h_1$ and $h_2$, respectively, from the lower and upper planes. Plate temperature $T$ varies only with time $t$ and coordinate $z$. Stress and strain tensor components are as follows:

\[
\begin{align*}
\sigma_x &= \sigma_x(x, t); & \sigma_y &= \sigma_y(x, t); \\
\sigma_z &= \tau_{xy} = \tau_{yz} = \tau_{zx} = 0; \\
\varepsilon_x &= \varepsilon_x(x, t); & \varepsilon_y &= \varepsilon_y(x, t); \\
\varepsilon_z &= \varepsilon_z(x, t); & \gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0.
\end{align*}
\]

From the strain compatibility condition
linear distribution of strain across $z$ results.

In order to apply plastic flow theory to the question of the stress-strain state of a plate it should be resolved into increments. In accordance with linear strain distribution

$$\frac{\partial^2 \varepsilon_z}{\partial z^2} \varepsilon_\phi = \frac{\partial^2 \varepsilon_z}{\partial z^2} = 0 \quad (1)$$

where $\varepsilon_{0x}$, $\varepsilon_{0y}$, $\varepsilon_{0z}$ are strain increments of the initial surface and its curvature. We present strain increments in the form of the sum of elasticity, plasticity, creep, and thermal expansion components:

$$\varepsilon = \varepsilon + \varepsilon + \varepsilon + \varepsilon^T, \quad (xy) \quad (2)$$

Here and subsequently the missing relationships are obtained by cyclic permutation of indices (symbol $(xy)$).

The elastic strain increment

$$\varepsilon = \frac{1}{E} (\sigma E^2 - \mu \sigma^0) - \frac{1}{E} \frac{\partial E}{\partial T} \left[ \left( \frac{\partial x}{\partial x} - \mu \frac{\partial y}{\partial y} \right) + \frac{\partial y}{\partial x} \frac{\partial x}{\partial y} \right] dT, \quad (xy) \quad (3)$$

where $E = E (z, T); \mu = \mu (z, T)$ are elasticity modulus and Poisson's ratio.

Thermal expansion

$$\varepsilon = \alpha = \alpha dT, \quad (5)$$

where $\alpha = \alpha (z, T)$ is the linear expansion coefficient.

The plastic strain increment $[3, 4]

$$\varepsilon = (F_0 + F_1 dT)(\sigma - \sigma), \quad (xy), \quad (6)$$

where $\sigma = (\sigma + \sigma) / 3$ is mean stress.

Functions $F_0 = F_0 (\sigma, T)$ and $F_1 = F_1 (\sigma, T)$ are

$$F_0 = \frac{3}{2} \{ \frac{1}{E} - \frac{1}{E} \}; \quad F_1 = - F_0 \frac{\partial \sigma}{\partial T}, \quad (7)$$

where $\sigma = \sigma (\varepsilon_{0i}, T)$ is the material instantaneous yield point; $\sigma$ is stress intensity; $\varepsilon_{0i} = f d \varepsilon_{0i}$ are accumulation rates for plastic strain increments (Odquist parameter); $E_k = E_k (\varepsilon_{0i}, T)$ is elastic shear modulus.

In order to account for creep strain it is possible to use flow theory or hardening theory $[5]$:

$$\varepsilon = \frac{3}{2} \frac{V_0}{\sigma_i} (\sigma - \sigma) \, dT, \quad (xy) \quad (8)$$

where $V_0$ is creep strain rate with uniaxial tension and constant stress $\sigma_0 - \sigma_1$, and at temperature $T$. 

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**Fig. 1.** Design layout.