AN ENERGY CRITERION FOR FATIGUE FAILURE

V. T. Troshchenko and P. A. Fomichev

UDC 539.388.1

A criterion for the fatigue failure of metals is presented which is based on specific energy dissipation during the loading cycle. Equations are obtained which determine the relative critical failure energy for cyclically unstable materials. The parameters used were studied as functions of the mean stresses. An experimental study of the criteria was carried out using regular and programmed loading on cyclically softening steels 45 and 12KhN3A in the as-received condition.

The establishment of fatigue failure criteria for metals undergoing high-cycle loading is one of the most important tasks of mechanics. In the mechanics of a deforming body three types of criteria are used — force, strain, and energy. As a force criterion during cyclic loading one can make use of a Weler curve which determines the number of loading cycles to failure as a function of the external load.

The strain criterion of fatigue failure is based on the Manson-Coffin function which relates the number of cycles to failure to the amount of plastic or total (elastic and plastic) deformation. The plastic strain per load cycle is taken into consideration along with the operating stress as well as the individual characteristics of the sample. In this context scatter in fatigue test results when constructing the endurance curve as strain versus number of cycles to failure is less than when constructing the curve as stress versus number of cycles to failure [1].

The energy criterion for fatigue failure is based either on the total amount of energy dissipated or on a separation of the total amount of energy dissipated into "unsafe" and "safe" components from the point of view of fatigue damage [2].

It was shown in [3] that during the high-cycle fatigue of most metals the total dissipated energy is strongly dependent on the number of cycles to failure, and in the general case can not be viewed as a fatigue failure criterion. This is true since it can not ensure that the energy criterion will be consistent over the whole region of the high-cycle loading.

It was mentioned [3] that the amount of unsafe energy dissipated per cycle $W'_u$ is obtained as a part of the total amount of dissipated energy $W'_t$:

$$W' = W'_t - W'_u,$$

where $W'_t$ is determined experimentally; $W'$ is the value sought; and $W'_u$ is an unknown function.

**Cyclically Stable Metals.** For cyclically stable metals the value of the total energy dissipated per cycle does not depend on the number of cycles to failure. The critical energy $W_{cr}$ dissipated over $N$ loading cycles before failure is

$$W_{cr} = (W'_t - W'_u) N,$$

from which

$$W'_u = W'_t - \frac{W_{cr}}{N}. \quad (2)$$

The function $W'_u (W'_t)$ is shown in Fig. 1. The second component in equation (2) corresponds to the cross-hatched ordinates in Fig. 1. At stress amplitudes equal to the fatigue limit $a_{ay}$, the number of stress cycles is infinite and $W'_{uy} = W'_t$. 

With increase in the stress amplitude the number of cycles to failure decreases and the second component in equation (2) increases as a power function. Since the total energy dissipated per cycle as a function of the stress amplitude follows a power law, equation (2) can be written in the following form:

$$W' = W - B \left( W_{u}^\alpha - W_{t}^\alpha \right),$$

(3)

where $\beta$ and $\alpha$ are the equation parameters; and $W_{t}^\alpha$ is the total energy dissipated per cycle at a stress amplitude equal to the fatigue limit of the metal. For metals which do not have a fatigue limit the equation has the following form:

$$W' = W - BW_{t}^\alpha.$$  

(4)

By substituting expressions (3) and (4) into equation (1) and using the differential form we obtain

$$\frac{dW}{dn} = R \left( W_{t}^\alpha - W_{t}^\alpha \right),$$

(5)

or

$$\frac{dW}{dn} = RW_{t}^\alpha,$$

(6)

where $n$ is the number of loading cycles; $R = B/W_{cr}$ is the equation parameter; $\tilde{W} = W/W_{cr}$ is the relative unsafe energy, which can be determined as the accumulated fatigue damage; and at failure $\tilde{W} = 1$.

The value of $W'_{t}$ sharply increases with increase in the stress amplitude. Therefore, variation in the results obtained for $\tilde{W}$, determined by using equations (5), (6), will only be considered near the fatigue limit.

The function (6) enables one to obtain an equation for the fatigue curve both in the Weler form $\sigma_{M}^{MN} = C_1$, and in the Coffin–Manson form $e_{ap}^{kN} = C_2$, where $e_{ap}$ is the mean value of half the hysteresis loop width. The total energy dissipated per loading cycle is equal to

$$W'_{t} = K_{ap} \sigma_{a} e_{ap},$$

(7)

where $K_{ap}$ is the shape coefficient of the hysteresis loop; $\sigma_{a}$ is the stress amplitude; and $e_{ap}$ is half the width of the hysteresis loop.