In the field of development of elements of the theory of fracture of composite solids an important place is occupied by investigations into the stress—strain state of brittle bodies weakened by sharp stress concentrators in the form of holes or inclusions with corner points (points of reentry) and cracks. From the viewpoint of application of methods of the mechanics of brittle fracture, the distribution of the components of the tensor of elastic stresses and the vector of displacements in the vicinity of the corner points of reentry, on the boundary dividing the two media, is of special interest.

At the present time most thoroughly investigated is the asymptotic distribution of stresses close to defects of the type of cracks [1-3], while the stress—strain state in the matrix close to sharp-cornered tips of absolutely rigid inclusions is as yet insufficiently studied, although investigations in this direction of the mechanics of fracture have already started [4].

On the basis of [4], the components of the tensor of elastic stresses in the vicinity of the corners of absolutely rigid inclusions can be represented in the following form:

\[ \sigma_x^{(i)} = \frac{1}{2V^2} \left( k_1^{(i)} \cos \frac{\beta^{(i)}}{2} \left( \alpha + 3 - 2 \sin \frac{\beta^{(i)}}{2} \sin \frac{3\beta^{(i)}}{2} \right) + \mathcal{O}(1) \right) \]

\[ \sigma_y^{(i)} = \frac{1}{2V^2} \left( k_1^{(i)} \cos \frac{\beta^{(i)}}{2} \left( \alpha + 3 - 2 \sin \frac{\beta^{(i)}}{2} \sin \frac{3\beta^{(i)}}{2} \right) + \mathcal{O}(1) \right) \]

\[ \sigma_{xy}^{(i)} = \frac{1}{2V^2} \left( k_1^{(i)} \sin \frac{\beta^{(i)}}{2} \left( 1 - \alpha - 2 \cos \frac{\beta^{(i)}}{2} \cos \frac{3\beta^{(i)}}{2} \right) + \mathcal{O}(1) \right) \]

where \( r, \beta^{(i)} \) are the polar coordinates with the origin at the corner of the inclusion and with a polar axis directed along the tangent to the surface of the inclusion; \( j \) is the ordinal number of the corner (the corners are numbered in the order of circulation, commencing with the positive direction of the Ox axis, counterclockwise); \( \alpha = 3 - \nu/1 + \nu \) for a plane stress state; \( \alpha = 3 - 4\nu \) for plane strain (\( \nu \) is Poisson's ratio). The quantities \( k_1^{(j)} \) and \( k_2^{(j)} \), called (as in the theory of cracks) the coefficients of stress intensity, depend on the applied loads and the parameters determining the configuration of the body and inclusions, as well as on Poisson's ratio of the material of the bonding agent. According to the data of [4], we have the relationship

\[ k_1^{(j)} - ik_2^{(j)} = 2 \frac{\varphi'(\zeta_{ex})}{\sqrt{\omega} \omega^*(\zeta_{ex})} \]

where \( \varphi'(\zeta) \) is the complex stress potential [5]; \( \omega^*(\zeta) \) is the second derivative with respect to \( \zeta \) of the function \( \omega(\zeta) \) which realizes conformal mapping of the exterior of a unit circle in the \( \zeta \) plane onto the exterior...
of the inclusion considered in the physical $z = x + iy$ plane; $\zeta_0$ are points of the unit circle in the $\zeta$ plane, corresponding, according to the transformation $z = \omega(\zeta)$, to the tips of the corner points of reentry on the boundary of the rigid inclusion; $\theta_j$ is the angle between the polar axis of the local $\tau$, $\beta^{(j)}$ coordinate system and the Ox axis.

The present report is a continuation of the investigations carried out in [4], where in the main the stress distribution was studied in the vicinity of the tips of sharp-cornered inclusions. Below an effective algorithm has been worked out for the calculation of the coefficients of stress intensity, $k_1^{(j)}$ and $k_2^{(j)}$, for a certain class of defects of the rigid inclusion type with points of reentry on the boundary.

As is known [5], the problem of stress–strain state of an elastic body with a rigid inclusion reduces to the solution of the second fundamental problem of the theory of elasticity for the region under consideration. One of the methods of solving fundamental problems of the theory of elasticity for regions with re-entrant points on the boundary is the method of going to a limit in the solution for a region which is bounded by a more general smooth contour, when this contour is stretched into one with a sharp end. In this way earlier [4] solutions were found to problems concerned with the stress–strain state of an elastic body with rigid rod or hypocycloidal inclusions.

However, for a fairly broad class of defects of the type of rigid sharp-cornered inclusions the process of calculating the coefficients of stress intensity can be considerably simplified.

We consider an infinite plane with a rigid sharp-cornered inclusion bounded by a contour whose exterior is transformed onto the exterior of a unit circle in the $\zeta$ plane by means of the mapping function

$$z = \omega(\zeta) = R \left( \zeta + \sum_{n=1}^{N} f_n^{(1)} \zeta^{1-m} \right),$$

where the coefficients $f_n$ are chosen so that the condition

$$\omega' = R (1 - \zeta^{-m}) Q_N(\zeta),$$

is satisfied. Here $Q_N(\zeta)$ is a polynomial all of whose zeros lie within the unit circle, and $m$ is the number of points of reentry on the boundary of the inclusion.

We refer the plane to a rectangular $xOy$ Cartesian coordinate so that the origin of coordinates coincides with the geometrical center of the inclusion, while the Ox axis passes in the direction of one of the reentrant points. Let such a plate be subjected to tension by uniformly distributed external forces $p$ and $q$ acting at infinitely remote points of the plate, with the forces directed at an angle $\alpha$ to the Ox axis. Under the action of the external forces, as well as owing to the rotation of the infinitely remote part of the plane, $\varepsilon_0$, the rigid inclusion is rotated about its center (the origin of coordinates) by a certain (as yet unknown) angle $\varepsilon$. The values of the displacement components on the boundary of the inclusion are here determined by the relationship

$$g = i\varepsilon \omega'(\sigma).$$

Thus, we arrive at the well-known second fundamental problem of the theory of elasticity [5] whose boundary condition on the contour of a unit circle in the $\zeta$ plane, in terms of the complex potentials $\varphi(\zeta)$ and $\psi(\zeta)$ has the following form:

$$-\frac{1}{\sigma} \frac{\partial}{\partial \sigma} \varphi'(\sigma) - \frac{1}{\sigma} \varphi'(\sigma) - 2\mu g\omega'(\sigma) = \omega'(\sigma) \psi'(\sigma).$$

Taking into account the fact that the principal vector and the principal moment of the forces acting from the side of the absolutely rigid inclusion on the plate are zero, as well as bearing in mind the character of the external load, on the basis of [5-8] the functions $\varphi(\zeta)$ and $\psi(\zeta)$ are sought in the form

$$\varphi(\zeta) = R \left[ \Gamma^\varphi + \sum_{n=1}^{N} a_n^\varphi \zeta^{1-n} \right];$$

$$a_n = \sigma_n + i\delta_n;$$

$$\psi(\zeta) = R \left[ \Gamma^\psi + \sum_{n=1}^{N} b_n \zeta^{1-n} \right];$$

$$b_n = \gamma_n + i\delta_n.$$