directions in which the specimens were cut; the $K_{1c}$ and $K_{IC}$ values obtained on Manjoin specimens and on specimens used in eccentric tension tests are practically identical.

In order to maintain the equality $K_{1C} = K_{IC}$, it is necessary that the stress system in the whole of the specimen on which $K_{1C}$ and $K_{IC}$ are determined and also the structural condition of the preliminary fracture zone are maintained constant.

LITERATURE CITED


CALCULATION OF THE STATIC CRACK PROPAGATION TRAJECTORY

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The problems linked with the determination of trajectories of quasistatic crack growth in brittle plates have usually been solved by variation methods [1, 2] and by the finite-element method [3-5]. Analytical approaches [6, 7] based on the solution of the problem of a slightly curved crack were also used. Here we describe the solution of this problem by a method of singular integral equations, which is extensively used in determining the stress-strain crack-bearing bodies.

Algorithm of the Solution of the Problem. Assume that a symmetrical plate contains an internal smooth crack of symmetrical or antisymmetrical shape. The centers of the crack and plate and their axes of symmetry coincide. The load applied is such that both crack ends grow in the same way, i.e., it is sufficient to describe the movement of only one of the crack tips. In the case of an edge or semi-infinite crack, the geometry of the region occupied by the body and the external load can be arbitrary. Assume that when the load reaches its limiting value the crack starts to move in a direction which forms with the tangent drawn to its tip the angle

$$\theta^* = \phi(K_I, K_{II}) \quad (1)$$

which is expressed in terms of stress intensity factors $K_I$ and $K_{II}$. Using conditions (1) the crack trajectory can be defined successively assuming that the initial crack has grown by a certain amount along the straight line in the direction of angle $\theta^* \ (1)$. Thereupon, after calculating the stress intensity factors for the new crack, we find the new value for angle $\theta^*$. Repeating the described procedure we obtain the crack trajectory in form of a broken line. This is the technique used in solving the problem by finite-element methods. However, in determining the stressed state of the body by the method of singular integral equations, the described approach is ineffective since the existence of a kink makes plotting of the solution very difficult. In addition, the experimental investigations of crack propagation (e.g., under cyclic load) show that the trajectory is actually a smooth curve. A kink can only exist at the very beginning of the original crack growth.
Fig. 1. Trajectories of a crack in a plate under uniaxial tensile stress.

Fig. 2. Stress intensity factors $K_I$ (solid lines) and $K_{II}$ (dotted lines) along the crack trajectory.

In [8] it was shown that the plane problem of elasticity theory for an arbitrary region with smooth curvilinear cracks leads to singular curvilinear equations which can be solved numerically. In order to use these results for plotting a trajectory, this must be approximated by a smooth curve. Let us place the origin of the rectangular coordinate system $xOy$ in the crack center with the $Oy$ axis directed along the symmetry axis if such exists. Assume that $x_0 > 0$ is the shape of the initial crack produced by the unique function

$$y = y_i(x), \quad 0 \leq x \leq x_1.$$  

Solving the problem in stages we shall approximate in each stage a section of the trajectory by a cubic parabola. Assume that in the $k$-th step the crack tip is determined by coordinates $x_k = x_i + (k - 1)h$ ($h > 0$, $k = 1, 2, \ldots$) and $y_k = y_k(x_k)$.

To obtain in the interval $[x_k, x_{k+1}]$ a convex (or concave) trajectory, we assume again that the crack ends in the point with abscissa $q_k = x_k - \delta_k$ ($\delta_k \geq 0$). Then, in the section $[q_k, x_{k+1}]$ the trajectory is described by equation

$$y_{k+1}(x) = a_{k+1}(x - q_k)^3 + b_{k+1}(x - q_k)^2 + c_{k+1}(x - q_k) + d_{k+1}.$$  

(2)

Let

$$y = \Gamma_k(x - x_k) + Y_k$$  

be the equation of a straight line passing through the tip $(x_k, y_k)$ in the initial crack propagation direction, i.e., $\Gamma_k = \tan(\alpha_k + \theta_k); \tan \alpha_k = y_k^\prime(x_k); \theta_k = f(K_{II_k}, K_{III_k})$, where $K_{II_k}$ and $K_{III_k}$ are stress intensity factors for the crack ending in the point $(x_k, Y_k)$.

From the condition that $x = q_k$ the curve $y = y_{k+1}(x)$ is a smooth extension of curve $y = y_k(x)$ and at point $x = x_{k+1}$ it smoothly links up with the straight line Eq. (3), yielding

$$c_{k+1} = y_k^\prime(q_k) = y_k^\prime; \quad d_{k+1} = y_k(q_k) = y_k;$$

$$a_{k+1} = \frac{(y_k + \Gamma_k(h + \delta_k) + 2(y_k - Y_k - \Gamma_k h))}{(h + \delta_k)^3};$$

$$b_{k+1} = \frac{3(Y_k - y_k + \Gamma_k h - (\Gamma_k + 2y_k)(h + \delta_k))}{(h + \delta_k)^3}.$$  

Assuming that parameter $\delta_k$ is small and requiring that the point of kink of curve (2) is at $x = x_{k+1}$, i.e., at $y_{k+1}'(x_{k+1}) = 0$, we obtain $\delta_k = h/2$ for $\Gamma_k \neq y_k^\prime$. For $\Gamma_k = y_k^\prime$ and $\delta_k = 0$ the parabola equation (2) degenerates to an equation of straight line, i.e., at $\theta_k = 0$ the trajectory coincides with the tangent drawn to the crack at its tip. Consequently, we have

$$\delta_k = \frac{h}{2}, \quad \Gamma_k = \Gamma_{k-1}, \quad \delta_i = \frac{h}{2}, \theta_i \neq 0,$$

$$0, \quad \Gamma_k = \Gamma_{k-1}, \quad h \gg 2; \quad \delta_i = 0, \theta_i = 0.$$  

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