SUMMARY. This approach does not define a probability measure by syntactical structures. It reveals a link between modal logic and mathematical probability theory. This is shown (1) by adding an operator (and two further connectives and constants) to a system of lower predicate calculus and (2) regarding the models of that extended system. These models are models of the modal system $S_5$ (without the Barcan formula), where a usual probability measure is defined on their set of possible worlds. Mathematical probability models can be seen as models of $S_5$.

Key words: Probability measure, axioms of Kolmogoroff, Choquet-capacity, upper- and lower probabilities, lower predicate calculus, modal lower predicate calculus, system $S_5$ of modal lower predicate calculus, Kripke-models of systems of modal lower predicate calculus.

1. INTRODUCTION

This paper attempts to link two formalisms: on the one hand a system of modal lower predicate calculus (mlpc) and on the other hand mathematical probability theory. This is first done without assuming any "interpretation" of probability. Further, it will be argued that a probability measure is not the entity to be interpreted directly, neither objectively nor epistemically; the genuine place of interpretation will turn out to be the set of possible worlds in a framework of (models of) mlpc. The formal aspects of the procedure may fit in either interpretation, an epistemic as well as an objective one. Nevertheless, it should be stressed that an objective interpretation will lead to a propensity interpretation, not to a frequency interpretation. The frequency aspects of such an interpretation are seen as consequences of the formalism (cf. Giere (1973, 1976), Niiniluoto (1981, 1988), Popper (1959)).

The connection between the two formalisms, mlpc and mathematical probability, is achieved by starting with a nonmodal system of lower predicate calculus (lpc), adding a one-place operator, $p$, which shows similar features as a probability measure, and two further connectives and constants. The operator $p$ and the two connectives are interpreted within the extended models of that lpc-system. This extension is straightforward. The main part of these new models will be a mapping whose domain is a system of sets entailing the images of the predicates under the interpretation. This mapping defines uniquely a probability measure on the $\sigma$-field defined on that system of sets, which can be formulated as a probability measure on the set of possible worlds of a model of modal lpc. The interpretation from the modal system, the associated modal model, is a simple extension of the inter-
pretation of the non-modal lpc. The specific accessibility relation on the
set of possible worlds is a consequence of the rules and axiom-schemata
concerning the operator $p$ added to the system of lpc. It will turn out
that the modal system achieved by this extension is imbedded in a modal
system $S_5$ (without the Barcan formula) of mlpc. Therefore the accessibility
relation on the set of possible worlds is an equivalence relation. On the
other hand, starting from “usual” mathematical probability models, these
models can be regarded as (the main) parts of models of mlpc.

Before starting with the elaboration of the program, it should be mentioned
what is not done in the present paper: In this paper neither a specific
probability measure is generated or “defined” by relations of the calculus
of modal logic nor a specific probability measure, as, e.g., a uniform
distribution, is constructed by means of syntactical relations. For those
points confer Bigelow (1976, 1977) and Giere (1976). In the present approach,
on the side of a logical system, an operator $p$ is defined, which shares
some features of a probability measure. And, on the side of the models
of that system, a usual probability measure may be deduced as “interpreting”
instance of that operator $p$. This probability measure is a genuine part
of the model and “varies” from model to model. This reflects the usual
practice using mathematical probability theory: The special probability
measure involved is determined by the situation investigated. It is not held
that a specific logical situation yields a specific probability measure (e.g.
a uniform) by virtue of its syntax.

The present formulations are in terms of finite additivity. This does not
mean that a generalization to $\sigma$-additivity is not possible, such a gene-
ralization may indeed be readily achieved. The specialization is only for
demonstrative purposes.

2. MOTIVATING REMARKS

Problems of probability arise not in the domain of mathematical probability
calculus but in formalizing “situations of the real world” by mathematical
probability. The questions are: What are those situations, how to speak
about them and to which features of those situations do the entities of
mathematical probability (as measures, random-variables, $\sigma$-fields and so
on) correspond? Concerning the situations there are two main views to
be distinguished: the objective or physical view on the one hand and the
subjective or epistemic view on the other hand.¹

(1) The objective view is faced with some well-defined (random) set-up
as, e.g., throwing a dice, spinning a roulette wheel or with the decay of
a certain atom, the momentum and the place of a particle. Those set-ups
produce observable instances which may show several values. The range
of the values which may be displayed by the set-up is determined and
well-defined in advance (the range of the values is called sample-space in
usual statistical practice), but it is not determinable in advance which value