The strains in components working at elevated temperatures are often reduced by means of cooling. Because of design problems and the need to reduce the weight of the structure, the amount of coolant is limited and, consequently, the amount of heat which can be carried away from the structure is also restricted.

Owing to the fact that the strength and stiffness of many materials depend greatly on the temperature, it is interesting to examine the problem of selection of temperature conditions in which in time \( t_0 \) the creep strain is minimum \( (\varepsilon_0) \) at the given amount of heat \( Q \).

We shall examine an idealized case of the uniaxial deformation of a thin rod (or a thin-walled pipe). It is assumed that the variation of temperature along the length of this component is insignificant, and the mean temperature \( t(t) \) will be introduced.

At the mass \( m \) of the component and the heat capacity \( c \) of its material, the amount of heat removed within time \( t_0 \) is equal to

\[
Q = \int_0^{t_0} c m \left[ t_0 - t(t) \right] dt,
\]

where \( t_0 \) is the nominal temperature without cooling (in a partial case, this temperature can be constant or increase, if heat sources are available). Equation (1) yields the condition

\[
\int_0^{t_0} b t(t) dt = a,
\]

where \( b = cm \) is a given time function;

\[
a = \int_0^{t_0} bt(t) dt - Q.
\]

If the cooling surface \( S \) and the heat-transfer coefficient \( \alpha \) are given,

\[
Q = \int_0^{t_0} \alpha S [t(t) - t_c] dt,
\]

where \( t_c \) is the temperature of the cooling medium.

Condition (2) is also obtained in this case (at \( b = \alpha S \))

\[
a = \int_0^{t_0} \alpha S dt.
\]

In accordance with [1], the dependence of strain rate \( \dot{\varepsilon} \) on stress for the uniaxial stress state has the form

\[
\dot{\varepsilon} = \frac{\sigma}{E} + \varphi_1(\sigma, t) \sigma + \varphi_2(\sigma) \varphi_3(t),
\]

where \( E \) is Young's modulus; \( \varphi_1 \), function of stress and temperature and is equal to zero at \( \sigma < 0 \); \( \varphi_2, \varphi_3 \), functions of stresses and temperature determined usually on the basis of experimental data.

Functional \( \varepsilon_0[t(t)] \) is given by the equation

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\[ e_0 = \int_0^t \left[ \frac{\dot{\sigma}}{E} + q_1(\sigma, t) \dot{\sigma} + q_2(\sigma) \varphi_3(t) \right] dt \]

at condition (2); consequently, the following functional must be examined as the extremum

\[ I = \int_0^T \left[ -\frac{\dot{\sigma}}{E} + q_1(\sigma, t) \dot{\sigma} + q_2(\sigma) \varphi_3(t) - \lambda \beta t \right] dt, \tag{4} \]

where \( \lambda \) is a constant quantity.

Euler's equation at the given function \( \sigma(t) \) determines the solution. If we are interested only in the creep strain in long-term service, then, disregarding elastic and plastic strains (the first and second term in Eq. (3)), we obtain

\[ \frac{d\varphi_3(t)}{dt} = \frac{\lambda b}{q_2(\sigma)}. \tag{5} \]

For example, at \( \varphi_3 = \exp (p t) \), where \( p \) is a constant \([2]\),

\[ t(t) = \frac{1}{p} \ln \left[ \frac{\lambda b}{q_2(\sigma)} \right]. \tag{6} \]

At low values of \( \sigma \) Eq. (6) gives \( t > t_0 \), i.e., it would be advantageous to heat the component at low stresses to enable faster cooling at higher stresses thus fulfilling condition (2). But for the majority of problems these conditions are of no practical interest because the cooling process in particular is important. Therefore, it should be stipulated that \( t(t) \leq t_0 \), and the extreme conditions will consist of sections of the extremal (5), (6) and of arcs of line \( t_0(t) \). The points of intersection of these lines determine the starts of the variation of the temperature conditions (e.g., start of cooling). For the case in which \( t_0 = \text{const} \), \( b = \text{const} \), Eq. (6) gives

\[ t(t) = \frac{a}{b t_0} - \frac{1}{p} \ln \left[ q_2(\sigma) \right] + \frac{1}{p r_0} \int_0^t \ln [q_2(\sigma)] dt. \tag{7} \]

An example will now be given of calculations for \( \sigma(t) \) varying in accordance with the linear law \( \sigma = \sigma_m(t/t_0) \) to the maximum value equal to \( \sigma_m \), at \( t_0 = 1000^\circ \mathrm{C} \).

If the hardening in creep is expressed in the form

\[ e^{\sigma} = \varphi_2(\sigma) \exp (p t) \]

or

\[ \varphi_2(\sigma) = \left( \frac{\sigma}{\sigma_0} \right)^n, \]

where \( \sigma_0 \), \( n \), \( m \), \( p \) are constants, the following functional is obtained instead of (4)

\[ I_1 = \int_0^T \left[ \exp (p t) \left( \frac{\sigma}{\sigma_0} \right)^n - \lambda \beta t \right] dt; \tag{8} \]

\[ t(t) = \frac{a}{b t_0} - \frac{p}{n} \ln \frac{n}{t_1} \text{ at } t > t_1; \]

\[ t(t) = t_0 \text{ at } t \leq t_1, \]

where \( t_1 \) is the start of cooling determined from Eq. (2) using the expression

\[ \frac{t_1}{t_0} + \ln \frac{t_1}{t_0} - 1 = \frac{\rho Q}{nb t_0}. \]