SENSITIVITY OF MATERIALS TO STRESS CONCENTRATION UNDER CYCLIC LOADING

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In calculating the strength of cyclically loaded machine parts operating in stress concentration conditions one needs a simple criterion of the sensitivity of material to stress concentration which would be independent of the concentrator parameters (radius at the notch tip \( r_n \), depth of notch \( t_n \), and the theoretical stress concentration coefficient \( \alpha_{\sigma} \)).

The Tamm – Buchmann criterion now used

\[ q = \frac{K_{\sigma} - 1}{\sigma_{\sigma} - 1} \]

(where \( K_{\sigma} \) is the effective stress concentration coefficient for the given material) varies between 0 and 1 depending on the concentrator dimensions [1, 2]. Using this criterion in strength calculations results in a considerable error.

I. A. Oding [3], on the basis of the cyclic strain curve equation represented by two straight lines \( \sigma = E \delta \) and \( \sigma_w = \text{const} \) (assuming perfect plasticity), derived a criterion for the sensitivity of material to a notch:

\[ v = \frac{\Delta \sigma_w}{\sigma_w} E, \]

where \( \Delta \sigma_w \) is the initial width of the tension – compression hysteresis curve corresponding to fatigue limit \( \sigma_w \), and \( E \) is modulus of longitudinal elasticity of the material.

Although the criterion \( v \) is a characteristic of the material, it cannot be used to find \( K_{\sigma} \) for different notches.

To derive a criterion of sensitivity of a given material to notches which would meed the above-mentioned demands, we used the cyclic strain equations

\[ \sigma_x = \sigma_w \left[ 1 - \left( 1 - \frac{\delta_x}{\delta_w} \right)^{\frac{1}{b}} \right], \]  

(1)

where \( \sigma_w \) and \( \delta_w \) is the fatigue limit of the material in case of tension and compression and the corresponding total deformation, and \( b \) is a parameter depending on the plasticity of the material: \( 0 < b = \sigma_w/\sigma_{\text{max}} < 1 \).

As seen in Fig. 1 (right side),

\[ \delta_w = \frac{\sigma_w}{b \xi}. \]

The curves described by (1) are in good agreement with the experimental hysteresis loops for materials with linear and soft characteristics [4].

According to the equation of Afanas'ev [5] transformed to the form used by Oding [3] (left side of Fig. 1), the cross-sectional stress distribution in a notched sample under tension – compression is given by

\[ \sigma_x = \sigma_0 \left( C \frac{A}{B} \right), \]  
where \( \sigma_x = \sigma_{\text{max}} / \sigma_0 \) is the nominal tension – compression stress of a flat notched sample; A, B, C are coefficients depending on notch parameters [3], and \( \sigma_0 \) is a theoretical stress concentration coefficient of a notched sample under tension.

The deformations are related to elastic stresses by the Hooke law:
\[ \delta_x = \frac{\sigma_x}{E}. \]  

In the presence of elasticplastic deformation the cross-sectional stress distribution in a notched sample can be found from (1) using expressions (2)-(4):
\[ \sigma_x = \sigma_\infty \left\{ 1 - \left[ 1 - \frac{1}{\sigma_0} \left( C \frac{A}{B} \right) \right] \right\} \frac{\delta}{\delta}. \]  
The fatigue limit of the notched sample is given by
\[ \sigma_n = \frac{1}{F} \int_{F} \sigma_x \, dF. \]  
The effective stress concentration coefficient is
\[ K'_{\sigma} = \frac{\sigma_\infty}{\sigma_n}, \]  
whence
\[ K'_{\sigma} = \frac{\int_{F} \left\{ 1 - \left[ 1 - \frac{1}{\sigma_0} \left( C \frac{A}{B} \right) \right] \right\} \, dF}{\sigma_\infty}. \]  

The values of \( K'_{\sigma} \) calculated from (7) are in good agreement with the experimental values of \( K_\sigma \) only for \( \alpha_\sigma < 2-3 \). At higher values of \( \alpha_\sigma \) the effective stress concentration coefficient \( K'_{\sigma} \) exceeds the experimental values (black dots in Fig. 2). Such a discrepancy can be explained by a rapid rise of the stress gradient at the notch surface with increasing \( \alpha_\sigma \). For \( \alpha_\sigma > 2 \) one has to allow for the effect of the stress gradient on the effective stress concentration coefficient.

Fig. 1. Distribution of tension – compression stresses along the cross section of a notched sample: 1) elastic deformation; 2) elasticplastic deformation; 3) cyclic strain as given by Eq. (1).

Fig. 2. \( K_\sigma \) vs \( \alpha_\sigma \) curves for tension – compression loaded notched flat samples of No. 10 steel: 1) according to Eq. (7); 2) according to Eq. (20); 3) according to Eq. (10) and data of [2].