A relationship for describing the kinetics of short fatigue cracks is proposed and experimentally confirmed on the basis of the energy two-parameter criterion of fracture. It is established that with use of this relationship it is possible to quantitatively describe the influence of the concentration and asymmetry of the stress cycle and the role of the compressive portion of the cycle and the surface layer on the kinetics of a crack and the threshold values $\Delta K_{th}$.

On the basis of the energy determination of the surface energy as the work expended on the creation of a free surface, as an engineering approximation it is assumed that the conditional surface energy at an internal point (imaginary surface) of a loaded uniform and isotropic elastic continuum is a function of the density of energy of deformations $W = \int_{ij} \sigma_{ij} e_{ij}$ to the nominal stresses $\sigma_{ij}$ at the given point:

$$\gamma = \gamma_0 (1 - \frac{W}{W_f}),$$  \hspace{1cm} (1)

where $\gamma_0$ is the surface energy of the unloaded body and $W_f$ is the critical energy density of the material without defects corresponding to fracture.

From Eq. (1) follows the energy criterion of fracture for an elastic (linear or non-linear) body with a crack [1, 2] (the indices of criticality are dropped):

$$\frac{W}{W_f} + \frac{J}{J_c} = 1,$$  \hspace{1cm} (2)

where $J$ and $J_c$ are the critical values of the J-integral of a body with a crack and its limiting (true) value for a body with an infinitely long crack, respectively.

Criterion (2) makes it possible to quantitatively describe the relationship of the characteristics of crack resistance to crack length and the level of nominal stresses [2].

In particular, for a body with a linearly exponential approximation of the strain curve $\bar{\sigma} = \bar{\varepsilon}^n$ in uniaxial tension criterion (2) has the form [2]

$$\frac{2\bar{\sigma}^{(1+n)/n} + \bar{\sigma} - 1}{2\bar{\sigma}^{(1+n)/n} + n - 1} + \frac{1}{n+1} \left( \frac{K_c}{\bar{K}_c} \right)^2 \bar{\sigma}^{(1+n)/n} = 1,$$  \hspace{1cm} (2a)

where

$$\bar{\sigma} = \sigma / \sigma_y; \quad \bar{\varepsilon} = e / e_y; \quad \bar{\sigma}_f = \sigma_f / \sigma_y;$$

$$\bar{\varepsilon}_n = \left\{ \begin{aligned} 1 \text{ with } \bar{\sigma} \leqslant 1, \\ n \text{ with } \bar{\sigma} > 1; \\ \end{aligned} \right.$$ 

$$\bar{K}_c = \sqrt{J_c} \cdot E'; \quad e_y = \sigma_y / E; \quad$$

$$\sigma_y = [\sigma_{0.2} / (E \cdot 0.002 + \sigma_{0.2})^{1/(1-n)}];$$
n is the strain hardening exponent in the elastoplastic area, and \( \sigma_f \) is the true tensile strength.

At present to describe the kinetics of fatigue cracks the most widely used are Paris type relationships:

\[
d\alpha/dN = C(\Delta K - \Delta K_{th})^m,\]

where \( C, m, \) and \( \Delta K_{th} \) are parameters of the material dependent upon crack length \( \alpha \), stressed and strained state, and the stress ratio of the loading cycle \( R \). With respect to this criterial relationship it is necessary to make the following remarks.

1. A Paris type relationship has a force character and therefore is not in a condition to completely simulate the energy expending mechanism of accumulation of damages by cyclic plastic deformations at a crack tip.

2. This relationship is of dimensional form and therefore does not possess the appropriate properties of invariance.

Based on the energy criterion (2) we may obtain an expression [2] for describing the cyclic crack resistance diagram without the above shortcomings:

\[
\frac{da}{dN} = S \left( \frac{\Delta J}{J_c} \right)^p,
\]

where

\[
\Delta J = J_{\text{max}} - J_{\text{th}} - \Delta J_{\text{th}};
\]

\[
J_{\text{max}} = \frac{J_{\text{max}}}{J_c} + \frac{W}{W_f};
\]

\[
J_{\text{th}} = \max \left( J_{\text{th}} \right) + \text{sign}(R) \cdot \frac{W_{\text{th}}}{W_f};
\]

\[
J_{\text{min}} = \max \left( J_{\text{min}}; J_{op} \right) + \text{sign}(R) \cdot \frac{W_{\text{min}}}{W_f};
\]

and \( J_{\text{max}} \) and \( J_{\text{min}} \) are the values of the J-integral corresponding to the maximum and minimum load (not to be confused with the cyclic J-integral).

Equation (3) for a body with a linearly exponential approximation of the strain diagram acquires a form similar to criterion (2a).

Let us consider a series of particular cases:

1) \( \bar{\sigma} \leq 1, 0 \leq \alpha < \infty \):

\[
\frac{da}{dN} = S \left( \frac{\tilde{K}_{\text{max}} - \tilde{K}_{\text{min}} - \Delta K^2_{\text{th}(R=0)}}{K_c^2} \right)^p;
\]

where

\[
\tilde{K}_{\text{max}} = \frac{K_{\text{max}}}{K_c} + \frac{W_{\text{max}}}{W_f};
\]

\[
\tilde{K}_{\text{min}} = \max \left( K_{\text{min}}, K_{op} \right) + \text{sign}(R) \cdot \frac{W_{\text{min}}}{W_f};
\]

2) \( \bar{\sigma} \leq 1, \alpha \rightarrow \infty \):

\[
\frac{da}{dN} = S \left( \frac{K_{\text{max}} - \max (K_{\text{min}}, K_{op}) - \Delta K^2_{\text{th}(R=0)}}{K_c^2} \right)^p;
\]