AN ANALYSIS OF METHODS OF DETERMINATION OF STRAIN USING DIVIDING GRIDS

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Various methods of calculation of the components of the tensor of strains $\varepsilon_{ij}$ from the displacements of the nodes of the dividing grid $U_{ij}$ are analyzed, including by the method of approximation by polynomials of the power $n = 1-5$, smoothing of $U_{ij}$, and approximation of polynomials of the power $n = 5-7$ and by methods of finite elements and numerical differentiation. A comparison of the calculated data with the results of experimental measurement of the field of local strains in Mg-1.5% Mn-0.3% Ce alloy with a grain size of 100 $\mu$m using a grid with a base of 50 $\mu$m showed that the method of finite-element approximation most adequately reflects the real picture of distribution of strains $\varepsilon_{ij}$. Statistical parameters characterizing the field of $\varepsilon_{ij}$ are discussed.

One of the methods most frequently used in experimental investigations for determination of the field of strains is the dividing grid method [1-3]. The components of the tensor of strains $\varepsilon_{ij}$ are calculated using the experimentally measured displacements of the nodes of the grid $U_{ij}$ applied before deformation by scribing [4], a photographic method [5], knurling, or other methods [6, 7] in the following manner

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right),$$

where $x_i$ are the coordinates of the nodes of the grid before deformation (Lagrange approach).

To determine the partial derivatives $\partial U_i/\partial x_m$ various numerical methods are used. From the mathematical point of view numerical differentiation is an unstable procedure, the result of which depends substantially upon the method of finding the derivatives [8]. This places researchers in a difficult position in selection of the method of determination of $\partial U_i/\partial x_m$ since little attention has been devoted to a comparison of the different methods from the point of view of their adequateness to the experimental data. In addition, determination of the statistical parameters most fully characterizing the field of strains is not sufficiently clear [9, 10].

The purpose of this work is an analysis of the different methods of finding of the components of the tensor of strains and of the statistical characteristics of the field of strains and comparing them with the values of $\varepsilon_{ij}$ determined directly from an experiment.

Material and Method of Measurement. The experiments were made on MA8 magnesium alloy (Mg-1.5% Mn-0.3% Ce). The original structure with a grain size of -100 $\mu$m and the surface using the specimens with a gage length of 20 x 6 x 2 mm were prepared using the earlier described method [11]. Using a PMT-3 hardness tester a square grid of scratches with a base of 200 $\mu$m was applied with a diamond needle. The specimens were deformed by tension on an Intron universal test machine at 670 K at a rate of $\dot{\varepsilon} = 4 \cdot 10^{-4}$ sec$^{-1}$. Upon reaching deformations of $\varepsilon = 3, 15, and 25\%$ the specimens were unloaded and the same area was photographed on an Epistand metallograph. The coordinates of the network nodes were determined from the photographs with a total magnification of 2000 x with use of a digitizer. The local strains were found from the equation

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right),$$

where $x_i$ are the distances between the nodes before and after deformation, respectively. The calculations were made on a Labtam computer and the data
Fig. 1. Deformation relief of MA8 alloy with $\varepsilon = 3\%$ (a), $15\%$ (b), and $25\%$ (c) and also the trajectories of movement of the nodes of the grid; $\ddot{\varepsilon} = 4 \times 10^{-4}$ sec$^{-2}$; $T = 670$ K (d).

obtained was presented graphically with use of Fortran-Graphor graphic expansion.

For calculation of the partial derivatives $\Delta U_k/\partial x_i$ the following were used:

1) approximation of $U_i$ by polynomials of the form [1, 2]

$$U_i = \sum_m \sum_n a_{mn} x_i^m x_j^n$$

with $r = n + m = 1 \ldots 5$;

2) smoothing of the field of displacements with subsequent approximation [1, 2] with $r = 5-7$;

3) linear approximation of $U_i$ using the method of finite elements [12];

4) determination of the derivatives using second power Lagrange interpolation polynomials [13].