In the limiting case of purely isotropic strengthening \( a_i = a_- = 0 \) and the boundary of the plastic zone will be described by Eqs. (8) and (9). In this case the yield strength \( \sigma_y \) entering here is a variable value, being a function of intensity of the accumulated plastic deformation \( \varepsilon \). Since regardless of the direction of loading trajectory isotropic strengthening is accompanied by an increase in \( \sigma_y \), the dimensions of the plastic zone will decrease in all directions.

Consequently in the general case of isotropic-kinematic strengthening the character of change in boundary of the plastic zone is determined by the ratio of the isotropic and kinematic constituents. If the kinematic constituent predominates, the form changes more significantly while if the isotropic constituent predominates, the dimensions contract.

**LITERATURE CITED**


**COMPUTING THE LIMITING STATE OF BANDED PIPES**

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The mathematical model of a rectilinear segment of a petroleum pipeline, banded on the outside by a high-strength section wound perpendicular to the generatrix of the pipe is proposed. The mathematical model accounts for stretching of the pipe, which occurs as the section is tensioned during fabrication of the arrangement. The equation and approximate analytical formula for a parameter of the pipe's stressed state are derived with allowance for preliminary stretching of the pipe and deformation due to internal pressure. The equation and recurrent relationship for calculating extremal internal pressure and deformation are obtained. The discrepancy between the computed data and experimental data obtained during full-scale tests of pipes 1220 mm in diameter is 2%.

Petroleum-transport problems, which have arisen in the last two decades, dictate the need to increase the strength and reliability of large-diameter pipelines. One of the most effective means of solving this problem is banding - the arrangement of a high-strength shape (for example, wire, fiberglass, woven glass fabric) in one or several layers on the surface of pipes, produced by industry using normal procedures, perpendicular to the generatrix of the pipe [1].

The basic concept of this construction is as follows: the forces in the structure are redistributed using a high-strength pretensioned winding so as to unload the pipe-base (make thinner) and to bring the efficiency of its operation to the level of a sphere due to equalization of longitudinal and circumferential stresses in the pipe; this will increase the strength and reliability of the oil pipeline.

Mathematical models of multilayer coiled thick-wall shells with allowance for the tension in the strip being wound have been investigated in sufficient detail [2, 3]. The mathematical model of thin-wall shells (for example, petroleum-grade pipes) should be sufficiently accurate in the elastoplastic zone of deformations to calculate the limiting values of the internal pressure; the effect of winding on the stressed state of the pipe-base is accounted for by the ratio of principal stresses, which is not a constant; the magnitude of the stretch in the pipe-base is used in the equations as a construction parameter.

These requirements are observed for the elastic region [4]; in the elastoplastic region, however, the model is insufficiently accurate due to piecewise-linear approximation of the strain diagrams.

The ratio of axial to circumferential stresses of a pipe banded with a wire is assumed constant and equal to 0.5; this does not correspond with reality [5]. The parameter of the pipe's stressed state is considered constant, and the thickness of the winding is defined as a function of the internal pressure, strains, and the parameter of the pipe's state; this leads to inadequate conclusions concerning the independence of the limiting pressure inside the pipe on the strength of the wire [6]. Further attempts to account for the effect of winding tension in the model [7] have made it possible to obtain only qualitative estimates of the strength, since no relationships between the stresses and tension in a pipe and other geometric construction parameters have been derived.

In our study, we propose a mathematical model of the strength of a wire-banded pipeline, which satisfies the above-formulated requirements.

Let us examine a thin-wall pipeline without axial deformation (underground installation), which is fabricated from an isotropic material and banded with wire. The wire material is also isotropic. A segment of the banded pipeline can be considered rectilinear and infinite (the effect of the ends of the pipe and the winding on the stressed state of the structure can be neglected). The number of layers and the density of the winding can be accounted for by its equivalent thickness. The conditions that we have enumerated enable us to use special conditions similar to those in [6-8] in the model. Let us designate all parameters that apply to the pipe and wire by the subscripts and superscripts \(p\) and \(w\), respectively.

The strain diagrams can be represented in an exponential approximation:

\[
\sigma_i^p = A_p\left(e_i\right)^{m_p},
\]

\[
\sigma_i^w = A_w\left(e_i\right)^{m_w},
\]

where

\[
A_p = \sigma_p^p\left(\frac{e_i}{m_p}\right)^{m_p}, \quad A_w = \sigma_w^w\left(\frac{e_i}{m_w}\right)^{m_w};
\]

\[
\varepsilon_i^{pl} = e_i^p - \frac{\sigma_i^p}{E_p};
\]

\[
\varepsilon_i^{wl} = e_i^w - \frac{\sigma_i^w}{E_w}.\]

The limiting state of the pipeline is attained in the region of elastoplastic deformations, and the elastic components in (3) and (4) can be neglected. Let us assume that the principal axes of the stresses and strains coincide with the geometric axes of the pipe, i.e., \(\sigma_1\) and \(\varepsilon_1\) are axial, \(\sigma_2\) and \(\varepsilon_2\) are circumferential, and \(\sigma_3\) and \(\varepsilon_3\) are radial stresses and strains, respectively.

Let us introduce the parameter of the pipe's stressed state

\[
k = \frac{\sigma_1^p}{\sigma_1^w}.\]

Using Sacks equation [9]