A procedure and analytical calculation are described for the thermal state of annular specimens with parabolic end surfaces necessary in order to choose the geometric shape and test schedules which provide excitation of prescribed thermal stresses. Values of temperature, thermal and mechanical bending, and contact stresses are prescribed which provide a specific type of material failure taking account of features close to holes and wedge tapers. Since specimen shape has some properties of the simplest multidimensional bodies it is possible to calculate the thermal state analytically.

In a previous communication [1] a test scheme is provided for thermomechanical fatigue together with a procedure for calculating the thermal state of annular specimens whose end surface is described by a parabola

\[ b = kr^{n^*}, \]

where \( b \) is specimen thickness; \( r \) is current radius; \( k, n^* \) are constants.

The stressed state of materials for test specimens is considered in the present work.

A test specimen was subjected to the thermal action of a gas stream over its toroidal inner surface and the mechanical action of rotating rolls on the outer cylindrical surface. The effect of the rest of the factors on stressed state is lower by several orders of magnitude. For example, centrifugal forces connected with specimen rotation are 300-3000 times lower than for turbine disks rotating with the same number of rotations per minute as a result of markedly lower specimen dimensions whose outer radius is 10-30 mm. The magnitude of thermodynamic forces with subsonic velocities of the gas stream does not exceed tenths of a percent of the basic load. An unfavorable effect on excitation of additional stresses is caused by vibration loads due to inaccurate specimen preparation and a loading of rolls which may reach several percent of the basic load.

Calculation of the stressed state for axisymmetrical bodies which differ in geometry from a cylinder or disk of constant thickness, and even for a constant thickness disk but with material properties varying over the radius, may be connected with use of approximate integral equations [2] or quite complex hypergeometric functions [3, 4]. Determination of the thermal stressed state of specimens with parabolic end surfaces is simplified with introduction of simple differential equilibrium equations [5]:

\[ \sigma_{\theta} = \frac{\partial}{\partial r} \left( \frac{\partial \left( \sigma_r r^{n^*} + 1 \right)}{r^{n^*}} \right), \]

where \( \sigma_{\theta}, \sigma_r \) are circumferential and radial thermal stresses respectively.

Taking account of equations for stress compatibility and the relationship between stresses and strains we obtain a fundamental differential equation

\[ \frac{\partial}{\partial r} \left( \frac{\partial \left( \sigma_r r^{n^*} + \frac{2}{n^*} \right)}{r^{n^*}} + E\alpha T \right) + n^* \frac{\sigma_r (1 + \mu)}{r} = 0, \]

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where $E$ is Young’s modulus; $\alpha$ is linear expansion coefficient; $T$ is temperature; $\mu$ is Poisson’s ratio.

After integration we obtain an equation for calculating thermal stresses

$$
\sigma_r = \frac{1}{r^{n+2}} \int_{r_1}^{r_2} \left( E\alpha T + n_\alpha \int_{r_1}^{r} \frac{\sigma_r (1 + \mu)}{r} \, dr \right) r^{n+1} \, dr - \int_{r_1}^{r} \left( E\alpha T + n_\alpha \int_{r_1}^{r} \frac{\sigma_r (1 + \mu)}{r} \, dr \right) r^{n+1} \, dr + \frac{C_1}{r^{n+2}} + C_2,
$$

where $C_1$ and $C_2$ are integration constants.

In the case of absence of external forces radial thermal stresses equal

$$
\sigma_r = 0 \text{ with } r = r_1 \text{ and with } r = r_2;
$$

$$
\sigma_r = \frac{1}{r^{n+2}} \left[ \left( n_\alpha + 1 \right) \frac{r_2^{n+2} - r_1^{n+2}}{r_2^{n+2} - r_1^{n+2}} \int_{r_1}^{r_2} \left( E\alpha T + n_\alpha \int_{r_1}^{r} \frac{\sigma_r (1 + \mu)}{r} \, dr \right) r^{n+1} \, dr - \int_{r_1}^{r} \left( E\alpha T + n_\alpha \int_{r_1}^{r} \frac{\sigma_r (1 + \mu)}{r} \, dr \right) r^{n+1} \, dr \right].
$$

By substituting an expression for calculating radial thermal stresses in the equilibrium equation we derive a calculation equation for estimating circumferential thermal stresses:

$$
\sigma_\theta = \frac{1}{r^{n+2}} \left[ \left( n_\alpha + 1 \right) \frac{r_2^{n+2} - r_1^{n+2}}{r_2^{n+2} - r_1^{n+2}} \int_{r_1}^{r_2} \left( E\alpha T + n_\alpha \int_{r_1}^{r} \frac{\sigma_r (1 + \mu)}{r} \, dr \right) r^{n+1} \, dr + \int_{r_1}^{r} \left( E\alpha T + n_\alpha \int_{r_1}^{r} \frac{\sigma_r (1 + \mu)}{r} \, dr \right) r^{n+1} \, dr \right] - \int_{r_1}^{r} \left( E\alpha T + n_\alpha \int_{r_1}^{r} \frac{\sigma_r (1 + \mu)}{r} \, dr \right) r^{n+1} \, dr - E\alpha T.
$$

With ratios of outer to inner diameter close to one radial stresses may be ignored or they may be found by a simple equation

$$
\sigma_r = \frac{1}{r^{n+2}} \left( \frac{\mu_n + 2 - \mu_1}{} \int_{r_1}^{r_2} \left( E\alpha T + n_\alpha \int_{r_1}^{r} \frac{\sigma_r (1 + \mu)}{r} \, dr \right) r^{n+1} \, dr - \int_{r_1}^{r} \left( E\alpha T + n_\alpha \int_{r_1}^{r} \frac{\sigma_r (1 + \mu)}{r} \, dr \right) r^{n+1} \, dr \right],
$$

With $r_2/r_1 > 2$ calculation is carried out by Eq. (1) using the method of successive approximations taking in the first approximation $\sigma_i = 0$.

It should be noted that with $r_2/r_1 > 2$ and $n_\alpha > 5$ it is technologically complicated to make a specimen due to the small thickness of the working edge, i.e. specimens with a ratio $r_2/r_1 < 2$ are most suitable for thermomechanical fatigue testing.

By using the functions $c_{\alpha}(\mu, r) \text{ and } S_{\alpha}(\mu, x)$, provides in [1] we write a solution of differential Eq. (1) in the following form:

$$
\sigma_r = \frac{E}{x} \left[ \frac{r^2 - r_1^2}{r_1^2 - r_2^2} I_n + \text{I} \right],
$$

$$
\sigma_\theta = \frac{E}{x} \left[ \frac{\nu r^2 - \nu r_1^2}{r_1^2 - r_2^2} I_n + \text{II} \right],
$$

where