The concept of the contact cross-section of a material produced from powder is absolutely indispensable but
does not suffice as yet for giving a complete analysis and understanding of powder compacting. The magnitude of
the contact cross section characterizes only the final degree of equilibrium attained in the compacting process, but
not the intermediary stages in this process. This parameter equals the cross-sectional area of solely the contact re-
gions which have remained intact during compacting, and never covers the areas of those contact regions which
were broken down or displaced in the course of the compacting process. Only in some cases may the difference be-
tween the limiting degree of contact and the actual degree (as determined from the porosity) give an indication as
to the magnitude of the broken and displaced contacts.

In addition to these theoretical considerations, there are some purely practical points which hamper the cal-
culation of the compacting process from the magnitude of the contact cross section. The main formula with which
this calculation is done can be written in the following three versions:

\[ \frac{P}{P_K} = \alpha; \quad P = P_0 \alpha; \quad P = \frac{P}{P_K \alpha}, \]

where \( P \) denotes the nominal pressure, \( P_K \) the contact compression pressure, \( \alpha \) the dimensionless contact cross-
section area expressed as a fraction of the nominal cross-sectional area.

This formula can be conveniently applied only in those pressure ranges where \( P_K \) is at least approximately
constant. However, the parameter \( P_K \) may vary from the Brinell hardness \( H_B \) of not cold-worked metal at a low
degree of compacting to the true metal strength \( H_A \) at zero porosity. According to the data of paper [2], the true
strength, or the energy capacity, as it is sometimes called, is equivalent to the energy needed for warming up a
unit volume of the metal to the reeking point, and completely melting this volume. Usually \( H_A / H_B \) equals about
12-20, i.e., \( P_K \) may vary more than tenfold during compacting, which makes it difficult to calculate the degree of
compacting from the parameter \( \alpha \).

We shall introduce the novel concept of the dimensionless volume fraction \( \omega \) of particles that are plastically
(irreversibly) displaced (deformed) during compacting. We then have:

\[ \frac{P_{K,d}}{P_{K,s}} = \frac{w}{\omega} = \frac{\Delta w}{\Delta \omega} = \frac{\int_{\omega=0}^{\omega} dw}{\omega} = \frac{\int_{\omega=0}^{\omega} P_{K,d} \, d\omega}{\omega} = \frac{P_{K,s}}{P_{K,s} \omega}, \]

where \( w \) denotes the reduced deformation work, i.e., the deformation work per unit particle volume \( \omega \), in other
words, the deformation work in an amount of powder with a weight equal to that of a unit volume of the compact
poreless material: \( w = \Delta w = \int_{\omega=0}^{\omega} dw; \quad \omega = \Delta \omega = \int_{\omega=0}^{\omega} d\omega; \quad P_{K,d} \) is the differential value of the critical stress
(contact pressure) and also the deformation work per unit of plastically distorted particle volume the moment
when \( w \) increases by \( dw \) and \( \omega \) by \( d\omega \) (when the deformation bears a local contact character \( P_{K,d} = P_K / \alpha \)); \( P_{K,s} \) denotes the total deformation work per unit volume of plastically distorted particles (from \( P = 0 \) at the beginning to \( P \) at the end of the process).

The \( P_{K,d} \) and \( P_{K,s} \) values are identical only if \( P_K = P / \alpha = \text{const} \). If \( P_K \) is not constant, then

\[
\frac{(P/\alpha)}{\text{init}} < P_{K,s} < \frac{(P/\alpha)}{\text{fin}},
\]

where \( (P/\alpha)_{\text{init}} \) denotes the initial, and \( (P/\alpha)_{\text{fin}} \) the final \( P_K \) value.

In the present paper it is shown that the final \( P_{K,s} \) values cannot exceed the initial values more than threefold. In deriving the equations and calculating the compacting processes it is more convenient to employ parameter \( \omega \) than parameter \( \alpha \). Parameter \( \omega \) always represent the sum of the plastically deformed volumes and covers both the volumes which adjoin the contact regions left intact at the end of the compacting process and those volumes which adjoin the regions in which the contact has been broken by rupture or displacement. Consequently, the value of parameter \( \omega \) directly reflects not only the result but also the history of the compacting process.

2. We shall now introduce the relationship between the relative density \( \delta \) and parameter \( \omega \) for the region of local plastic deformation near the contact (intermediary or second compression stage). This relationship is generally not univalent. Solely the relationship between the limiting \( \omega \) value (at a given porosity) and \( \delta \) is univalent. This limiting value may be calculated from the probability theory. We shall consider an ideal model of powder compacting from \( \delta = 0 \) to a given \( \delta \) value. To an increment \( d\delta \) of the relative density corresponds an increment \( d\theta \) of the volume fraction of plastically distorted material, which is proportional to the increment \( d\omega \), and the proportionality factor is the volume occupied by the particles:

\[
d\omega = \theta d\delta; \quad \omega = \int_0^\delta \theta d\delta = \frac{\theta^2}{2}; \quad \omega_0 = \frac{\theta_0^2}{2}
\]

where \( \theta_0 \) denotes the relative density with which the compacting process is actually started (\( \theta_{\text{init}} \leq \theta \leq \theta_{\text{shake}} \)); \( \omega_0 \) the initial limiting fraction of plastically deformed volume corresponding to the initial density \( \theta_0 \) before compacting; parameter \( \omega = \Delta \omega = \omega_\delta - \omega_0 = \frac{\theta^2 - \theta_0^2}{2} \). Consequently, the volume fraction equals to \( 1 - \theta_0 = \Pi_0 \), constitutes the void volume into which new particles are pushed during compression from \( \theta_0 \) to \( \theta \); for \( \delta \) and \( P_{K,s} \) see formulas (2) and (2a).

It is quite evident that formulas (4) are based on an arbitrary extrapolation of the regularities valid in the three actual compression stages (\( \theta \leq \delta \leq 1 \)) to the range \( 0 \leq \delta \leq \theta_0 \). For example Eq. (4) predicts that \( \omega_0 = 19\% \) at \( \delta = \theta_0 = 60\% \). The actual value of \( \omega_0 \) is lower by several powers of ten and almost equals zero.

Hence it follows that we have to start from a real compression scheme covering solely the range \( \theta_0 \leq \delta \leq 1 \). Compression of the initial volume of \( 1 / \theta_0 \) cm\(^3\) with \( \delta = \theta_0 \), to the final volume of \( 1 \) cm\(^3\) with \( \delta = 1 \) means that new particles are pushed into the cube of \( 1 \) cm\(^3\) capacity; the initial volume fraction occupied by particles equals \( \theta_0 \), the initial porosity is \( 1 - \theta_0 = \Pi_0 \) and the initial value of the deformation work \( w_0 = 0 \). Consequently, the volume fraction equals to \( 1 - \theta_0 = \Pi_0 \), constitutes the void volume into which new particles are pushed during compression, and the volume \( \theta_0 \) has to be left out of consideration when the probability of collision between the moving particles and the solid phase is determined. This probability will equal \( \Delta \theta = \frac{\Delta \omega}{\Pi_0} = \frac{\delta - \theta_0}{\Pi_0} \). Hence it follows that the limiting \( \omega \) value (at a given porosity) is given by:

\[
d\omega = \frac{d\omega}{P_{K,d}} = \frac{d\omega}{P_K} = \frac{d\omega}{P_{K,s}} d(\Delta \theta) = \frac{\Delta \theta}{\Pi_0} d\theta; \quad \Delta \omega = \omega = \frac{\omega}{P_{K,s}} = \frac{(\Delta \theta)^2}{2\Pi_0}; \quad \omega = \frac{P_{K,s}}{2\Pi_0}.
\]