THE COEFFICIENT OF LATERAL PRESSURE

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The value of the coefficient of lateral pressure $\xi$ has been the subject of numerous investigations. Bal'shin [1] has proposed that, like Poisson's ratio, the coefficient of lateral pressure should be regarded as being dependent on the density of a powder body ($\theta$):

$$\xi = \xi_m \theta,$$  \hspace{1cm} (1)

where $\xi_m$ is the coefficient of lateral pressure for the bulk material.

Bal'shin derives Eq. (1) by assuming that the lateral pressure is transmitted only through the metallic cross section of a cubic element. In such a case, it should not be forgotten that the vertical compaction pressure is also transmitted through a corresponding metallic cross section of this element. Equation (1), however, cannot be regarded as entirely valid from a theoretical point of view, and is not supported by experimental data. Doubts regarding the validity of Eq. (1) have been voiced by a number of investigators [2-5]. Experiments conducted at the Institute of Materials Science, Academy of Sciences of the Ukrainian SSR, have demonstrated that the Poisson's ratio of sintered iron compacts increases with rise in density [6]. Other findings [7] also provide qualitative support for Bal'shin's view that the coefficient of lateral pressure increases with rising density of a powder body.

In investigations into the pressing of iron powders [2, 3], the dependence of lateral pressure on axial was found to be virtually linear at $p = 0-4000 \text{ kg/cm}^2$. The value of the coefficient of lateral pressure $\xi$ established in these studies lay within the range 0.38-0.41. In another investigation [5], it was found that, over the compaction-pressure range from 300 to 3000 $\text{kg/cm}^2$, the coefficient of lateral pressure is 0.35-0.42 and exhibits a slight tendency to rise with increasing pressure and density.

Torkar [8] recommends the following relationship for determining the coefficient of lateral pressure:

$$\xi = \tan^2 \left(45^\circ - \frac{\rho_i}{2}\right),$$ \hspace{1cm} (2)

where $\rho_i$ is the angle of interparticle friction ($\rho_i = \arc\tan f_i$). This relationship is in satisfactory agreement with experimental data only at low compaction pressures.

Balhausen [9] considers that the coefficient of lateral pressure is approximately equal to the tangent of the angle of repose:

$$\xi \approx \tan \beta,$$ \hspace{1cm} (3)

This relationship, however, has not been verified experimentally. Radomysel'skii [10, 11], on the basis of his experiments, came to the conclusion that the value of the coefficient of lateral pressure is proportional to some power of the density of a powder body. It must be admitted, however, that this claim requires additional clarification.

Analyzing the foregoing, it may be concluded that, in view of the existence of a number of contradictions, an improved method for determining the coefficient of lateral pressure is required.

Let us consider a typical scheme of powder-particle reaction in the course of pressing, as illustrated in Fig. 1. We will assume that, at the instant of deformation under consideration, the compact is in the shape of a cube with a face area $S_n$. There is no external friction on the side faces of the cube.

The reaction forces exerted by the particles in contact with the side die faces may be expressed as:
Accordingly, the static-equilibrium condition for the cube face in contact with the end face of the punch will be given by the equation:

\[ P_x = \sum_{i=1}^{n_0} \sum_{i=1}^{K_a} a_i S_{ci} (\sin \varphi_i \cos \alpha_{xi} - f_i \sin \tau_i \cos \beta_{yi}); \]  
\[ P_y = \sum_{i=1}^{n_0} \sum_{i=1}^{K_a} a_i S_{ci} (\sin \varphi_i \cos \alpha_{yi} - f_i \sin \tau_i \cos \beta_{yi}); \]  

where \( \varphi_i \) and \( \tau_i \) are, respectively, the angles of normal and tangential force reaction of particle contacts, whose intensity is \( \alpha_{ci} \); \( \alpha_{xi} \) and \( \alpha_{yi} \), respectively, are the angles between the \( x-x \) axis and the vector \( \alpha_{ci} S_{ci} \sin \varphi_i \), and between the \( y-y \) axis and the vector \( \alpha_{ci} S_{ci} \sin \varphi_i \); \( \beta_{xi} \) and \( \beta_{yi} \), respectively, are the angles between the \( x-x \) axis and the vector \( \beta_{ci} S_{ci} \sin \tau_i \), and betwen the \( y-y \) axis and the vector \( \beta_{ci} S_{ci} \sin \tau_i \).

Changing over to mean statistical values and considering that the forces \( P_x \) and \( P_y \) are independent of the directions chosen for the \( x \) and \( y \) axes, it is necessary to employ the following relations:

\[ \alpha_x + \alpha_y = \frac{\pi}{2}; \quad \beta_x + \beta_y = \frac{\pi}{2}; \quad \beta_x = \beta_y = \frac{\pi}{4}. \]  

In this case, Eqs. (4), (5), and (6) assume the following form:

\[ P_x = \sigma_c S_{cs} n_a K'_a (\sin \varphi - f_i \sin \tau) \cos \frac{\pi}{4}; \]  
\[ P_y = \sigma_c S_{cs} n_a K'_a (\sin \varphi - f_i \sin \tau) \cos \frac{\pi}{4}; \]  
\[ P_z = \sigma_c S_{cs} n_a K'_a \cos \phi \cos \frac{\pi}{4}. \]  

where \( n_{ci} \) is the mean statistical number of particles in the cross section of the cubic element; \( S_{cs} \) and \( \sigma_c \), respectively, are the mean statistical values of contact surface area and contact-reaction pressure; \( K'_a \) is the mean statistical number of particle-support contacts.

Clearly, the coefficient of lateral pressure \( \xi \) is given by the ratio of the forces exerted by the powder body being pressed on the die side faces \( (P_x, P_y) \) to the active compaction force \( P_z \):

\[ \frac{P_x}{P_z} = \frac{P_y}{P_z} = \frac{\sin \varphi - f_i \sin \tau}{\cos \varphi - f_i \cos \tau} \cos \frac{\pi}{4}. \]