ANALYSIS OF ENERGY CRITERIA FOR FAILURE WITH LOW-CYCLE LOADING

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A study is made by experiment of the low-cycle fatigue of high-strength steel at 20, 450, and 600°C for steels of low (22K) and moderate (16GNM) strength when the amplitudes of specific energy per cycle of elastic, plastic, and elastoplastic deformation, and also values for similar energies accumulated for the whole specimen life are taken as physical criteria of failure. Analytical description of experimental low-cycle fatigue curves in an energy interpretation is carried out. It is shown that as physical criteria with low-cycle fatigue it is more desirable to take the amplitude of energy per elastoplastic deformation cycle accumulated for the whole specimen life.

The durability of a material is most often connected with force treatment of the failure process, i.e. with an increase in operating stress, or with a strain approach, i.e. with the level of plastic or elastoplastic strains [1, 2, etc.]. In addition, in a number of studies the process of damage accumulation in a material under multicycle loading conditions is considered from an energy viewpoint. Berenov [3] and other scientists [4] have suggested that the increase in overall dissipated hysteresis energy with multicycle loading equals material failure energy in the case of static breaking. Gol'tsev [3] recommended as a criterion of fatigue failure the difference between energy dissipation per cycle and a similar value at the fatigue limit. In systematic studies Troshchenko has shown that under multicycle loading conditions some combination of these energies should be used as a failure criterion [5]. Other studies are also known on this subject [6-8].

Energy criteria for failure in the low-cycle region have not been studied sufficiently. In view of this in the present work energy criteria for failure were and for studied with low-cycle loading of high-strength Cr-Ni-Mo steel at 20°C (σ₀₂ = 1240 MPa, σₜ = 1380 MPa, δ = 13%; ψ = 38%), 450°C (σ₀₂ = 790 MPa, σₜ = 952 MPa, δ = 12%, ψ = 58%) and for steels of low 22K (σ₀₂ = 150 MPa, σₜ = 312 MPa, δ = 23%, ψ = 85%) and for steels of 600°C (σ₀₂ = 246 MPa, σₜ = 458 MPa, δ = 33%, ψ = 69%) and medium 16GNM strength (σ₀₂ = 433 MPa, σₜ = 601 MPa, δ = 25%, ψ = 64%).

The failure criteria taken were the amplitudes of specific energy per cycle of elastic uₑ, plastic uₚ, and elastoplastic u strain in the stabilization stage for the cyclic elastoplastic deformation stage, and also amplitudes of similar energies uₑN, uₚN, uN accumulated for the whole specimen life.

In a procedural respect it is desirable to take as specimen life the number of cycles to formation of a crack of considerable size. On the basis of factor analysis of experimental results it is shown [9] that with low-cycle fatigue of steels at normal and elevated temperature the number of cycles Nₚ to formation of a crack 0.5-0.8 mm long and to failure N is connected by the relationship

\[ Nₚ = cₙN^{mₚ}, \]

\[ cₙ = 0.676 - 0.662 \times 10^{-3}T - 0.086σₛ + 0.825 \times 10^{-4}σₛ^2, \]

\[ mₚ = 0.990 + 1.22 \times 10^{-4}T + 0.030σₛ + 0.025 \times 10^{-4}σₛ^2, \]

where T is test temperature, °C; σₛ is theoretical stress concentration factor in the elastic region. Taking this into account and the well-known assumptions of Manson [10] low-cycle fatigue curves in amplitudes of elastoplastic strains ε and stresses σ may be presented depending on the number of cycles Nₚ in the following form:

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\[ \varepsilon = C_p \left( \frac{N_T}{c_N} \right)^{m_N} + \frac{C_e}{E_T} \left( \frac{N_T}{c_N} \right)^{m_N}; \]

\[ \sigma = C_\sigma \left( \frac{N_T}{c_N} \right)^{m_N}, \]

where \( C_p = \frac{(E_T/2E) \ln (100/(100 - \psi))}{10^6}; \ C_\sigma = 1.75 \sigma_T; E, E_T \) are elasticity moduli at normal and elevated temperatures; \( \psi \) is relative reduction of area in the neck with breaking at normal temperature; \( \sigma_T \) is ultimate strength at the temperature of low-cycle fatigue tests; \( p, e \) are parameters.

By excluding from relationship (4) the number of cycles \( N_T \), taking account of Eq. (5) we obtain a cyclic elastoplastic deformation curve in the stage of its stabilization:

\[ \varepsilon = \frac{\sigma}{E_T} + C_p \left( \frac{\sigma}{C_\sigma} \right)^{p/e} \]  

(6)

By drawing attention to Eq. (6) we find the specific potential energy of elastoplastic deformation:

\[ u_{ep} = \frac{\sigma^2}{2E_T} + \frac{p C_p C_\sigma}{p + e} \left( \frac{\sigma}{C_\sigma} \right)^{p/e+1}. \]  

(7)

If we bear in mind Eq. (5), then from the last relationship it is possible to obtain an equation for the low-cycle fatigue curve with an energy interpretation:

\[ u = \frac{C_\sigma^2}{2E_T} \left( \frac{N_T}{c_N} \right)^{m_N} + \frac{p C_p C_\sigma}{p + e} \left( \frac{N_T}{c_N} \right)^{m_N} - \frac{p + e}{m_N}, \]  

(8)

where \( u \) is the amplitude of specific elastoplastic deformation energy. The terms in Eq. (8) are amplitudes of elastic \( u_e \) and plastic \( u_p \) energy per cycle. Consequently, the equations of the corresponding low-cycle fatigue curves may be written in the form

\[ u_e = \frac{C_\sigma^2}{2E_T} \left( \frac{N_T}{c_N} \right)^{m_N} - \frac{2e}{m_N} ; \quad u_p = \frac{p C_p C_\sigma}{p + e} \left( \frac{N_T}{c_N} \right)^{m_N} - \frac{p + e}{m_N}. \]  

(9)

Taking this into account the values of specific energies for elastic, plastic, and elastoplastic deformation accumulated for the whole specimen life in relation to the number of cycles \( N_T \) are

\[ u_{eN} = \frac{C_\sigma^2 c_N}{E_T} N_T^{1-\frac{2e}{m_N}}; \]  

(10)

\[ u_{pN} = 2p C_p c_N \frac{p + e}{m_N} N_T^{1-\frac{p + e}{m_N}}; \]  

(11)

\[ u_N = \frac{C_\sigma^2 c_N}{E_T} N_T^{1-\frac{2e}{m_N}} + 2p C_p c_N \frac{p + e}{m_N} N_T^{1-\frac{p + e}{m_N}}. \]  

(12)

With the aim of experimental verification of the suitability of energy criteria for failure low-cycle fatigue tests were performed for plane specimens under pure bending conditions with a symmetrical deformation cycle. The test frequency was 8 cycle/min. Specimens of high-strength steel were heated by passing an electric current. It is established that for high-strength