MODELING OF INHOMOGENEOUS BIAXIAL THERMALLY STRESSED STATES OF MATERIALS IN THE RANGE OF PRINCIPAL-STRESS RATIOS FROM $-\infty$ TO $+\infty^*$

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A procedure for creating conditions for the failure of material at assigned temperatures, thermal stresses, and principal-stress ratios is described. An example of the analytical calculation of the thermal and stress states of the material in specimens is cited.

Structural components fail under heat exchange not only near tapered edges, but also at other points of thermal-stress concentration. As a rule, these are taper points in the wall thickness of hollow blades, in the disks of gas turbines, the bottoms of pistons in internal-combustion engines etc., in which a biaxial thermally stressed state is realized. This material state can be induced in the specimens proposed in [1], which assume the form of a ring with inner 1 and outer 2 cylindrical surfaces (Fig. 1). Some of the faces are built in the form of paraboloids 3 and 3', and the others in the form of hyperboloids 4 and 4' so that at a certain distance from the center of the specimen, its thickness is constricted to an assigned minimal value in the axial direction. Faces 3 and 3' and also 4 and 4' can be fabricated symmetrically. The more general case of the asymmetric shaping of the faces of the specimen is presented in Fig. 1. The generatrices of the faces are described by the equation

$$b = kr^n$$

where $b$ is the current thickness of the specimen in the axial direction, $r$ is the current radius, $k$ is an arbitrary positive coefficient, and $n_*$ is a coefficient, which has integer values, and is positive for the parabolic section, and negative for the hyperbolic section.

A thermally stressed state with a principal-stress ratio that varies from $\sigma_0/\sigma_r = -\infty$ on the inner surface to $\sigma_0/\sigma_r = +\infty$ on the outer surface along the radius is excited in the annular specimen with flat faces perpendicular to the axes when its inner surface is heated. This is associated with the fact that as is apparent from Fig. 2, the circumferential thermal stresses assume maxima on the inner and outer surfaces; in this case, they are of opposite sign, and are equal to zero at a certain distance from the center of the specimen between the inner and outer surfaces. The radial thermal stresses are, conversely, equal to zero on the inner and outer surfaces and attain a maximum between these surfaces.

Thus, the ratio of the circumferential thermal to the radial stresses varies from $-\infty$ to $+\infty$ along the radius of the specimen, using all possible combinations of principal stresses. Failure of the material frequently occurs, however, on the inner surface of the specimens indicated. This is caused by high maximum radial stresses, and also by the high temperature of the inner surface as the specimen is heated. The magnitude of the thermal stresses depends primarily on the temperature distribution along the radius of the annular specimen and is independent of the thickness of the ring in the axial direction, if this thickness does not vary along the radius. If, however, the thickness of the specimen varies along the radius, the minimum of this thickness always exerts a significant influence on the magnitude of the radial thermal stresses and changes the circumferential stresses negligibly. Varying the minimum of the specimen thickness and its position relative to the center of the specimen, consequently, it is possible to increase significantly the level of radial thermal stresses and to give rise to the material's failure at a given point in the specimen under a given combination of principal stresses and temperature.

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Fig. 1. Specimens for thermal-fatigue testing of materials in biaxial thermally stressed state.

Fig. 2. Distribution of circumferential (solid lines) and radial (broken lines) stresses along radius of hollow disk (1, 2) and specimen having minimum thickness at distance $r_{b_{\text{min}}}$ (3, 4).

The faces of the specimen can shaped in the form of any concave surface. For a specimen with faces in the form of paraboloids that narrow toward the center and hyperboloids that expand toward the center, the temperature and stress fields can be calculated exactly by analytical methods when the inner or outer surface is subjected to cyclic heating [2, 3].

The following heat-conduction equation for specimens with parabolic faces is cited in [2]:

$$\frac{1}{\rho^* + 1} \frac{\partial}{\partial r} \left( \rho^* + 1 \frac{\partial T}{\partial r} \right) a = \frac{\partial T}{\partial t},$$

where $r$ is the current radius of the specimen, $T$ is the current temperature of the specimen, $\rho$ is the thermal diffusivity, $\rho^*$ is the exponent in the equation of the generatrix of the face, and $t$ is time.