NONLINEAR VIBRATIONS OF SHELLS AND PLATES WITH ACCOUNT FOR ENERGY DISSIPATION IN THE LAYERS

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We solve the problem of the vibrations of shells and plates with large deflections with account for energy dissipation in the layers and analyze the influence of the structure of the packet of layers on the vibration decrement.

We examine a shallow shell, consisting of an arbitrary number $n$ of layers, working without separation or slip, of unspecified thickness $h_k = a_k - a_{k-1}$, density $\rho_k$, and stiffness (Fig. 1).

It is known that the results of solution of the problem of the vibrations of such structures depend significantly on the deformations of transverse shear and contraction. The use of the Kirchhoff–Love hypotheses may not be legitimate in this case. Therefore we propose to obtain the solution with account for the refining factors and show the degree of their influence on the results.

Assuming that the deflections are commensurate with the shell thickness, we construct the solution in the geometrically nonlinear formulation. In addition, we introduce a nonlinear relationship between the stresses and the relative deformations [1], which leads to the formation of a hysteresis loop. In this way we account for energy dissipation during the vibrations.

We reduce the three-dimensional problem of elasticity theory to the two-dimensional form on the basis of the hypotheses of the nonclassical theory of heterogeneous structures [2]. In contrast with [3], where the hypotheses were adopted for each layer, we shall write them for the entire packet. Then with account for the deformations of transverse shear and contraction we obtain by analogy with [1, 2] the following stresses in the $k$-th layer:

$$
\sigma^{(k)}_{ij} = \sigma_{ij}^{(k)} \pm \frac{3}{8} \sigma_{ij}^{(k)\text{max}} \delta_{ij}^{(k)} (1 \pm 2 \cos \theta - \cos^2 \theta) \quad (i, j = 1, 2);
$$

$$
\sigma^{(k)}_{33} = \sigma_{33}^{(k)} \pm \frac{3}{8} \delta_{33}^{(k)\text{max}} (1 \pm 2 \cos \theta - \cos^2 \theta),
$$

(1)

(2)

(3)

where the subscript max corresponds to the largest values of the stresses at the initial time ($t = 0$), the symbol $\rightarrow$ denotes the ascending branch of the hysteresis loop, and the symbol $\leftarrow$ denotes the descending branch; $\delta_{ij}^{(k)}, \delta_{ij}^{(k)\text{max}}, \delta_{33}^{(k)}, \delta_{33}^{(k)\text{max}}$ are the vibration decrements, depending respectively on $\sigma_{ij}^{(k)}, \sigma_{33}^{(k)}; \theta = \omega t + \varphi$ ($\omega$ is the frequency and $\varphi$ the phase shift of the vibrations).

On the basis of the Hamilton–Ostrogradskii principle, with the use of (1)-(3) we obtain the equations of motion:

$$
\ddot{N}_{\dot{y}, j} - U_i = 0;
$$

(4)

$$
\ddot{M}_{\dot{y}, y} - (K_1 N_{11} + K_2 N_{22}) - U_{\dot{y}, y} - U_3 + (q^+ + q^-) = 0;
$$

(5)

$$
\ddot{M}_{\dot{y}, y} - \dot{Q}_{\dot{y}, y}^{(1)} - U_{\dot{y}, y}^{(1)} = 0;
$$

(6)
Fig. 1. Scheme of studied shallow shell (1 is the layer number).

\[ M_{ij}^{(3)} - Q_{ij}^{(3)} - U_{ij}^{(3)} + q\varphi_{31}(a_0) + q^+\varphi_{3n}(a_n) = 0 \]  
\[ M_{ij}^{(3)} - Q_{ij}^{(3)} - U_{ij}^{(3)} + q\varphi_{31}(a_0) + q^+\varphi_{3n}(a_n) = 0 \]  
and the corresponding boundary conditions for the edge \( x_1 = \text{const} \):

\[
\begin{align*}
N_{ij} \delta U_1 &= 0; & N_{ij} \delta U_2 &= 0; & M_{ij} \delta w_1 &= 0; & (M_{ij} + 2M_{ij,2} - U_{ij}) \delta w &= 0; \\
M_{ij} \delta \chi_{1,1} &= 0; & (M_{ij} + 2M_{ij,2} - U_{ij}) \delta \chi_{1,1} &= 0; \\
M_{ij} \delta \chi_{2,1} &= 0; & (M_{ij} + 2M_{ij,2} - U_{ij}) \delta \chi_{2,1} &= 0; \\
M_{ij} \delta \chi_{3,1} &= 0; & (M_{ij} + 2M_{ij,2} - U_{ij}) \delta \chi_{3,1} &= 0.
\end{align*}
\]  

We can by analogy with (1)-(3) represent the forces and moments in (4)-(9) as consisting of two parts. One of them \( (N_{ij}, M_{ij}, Q_{ij}, Q_{ij,2}, M_{ij,2}) \), \( p = 1, 2, 3; Q_{ij,2}, s = 2, 3 \), corresponds completely to [3], while the other describes the imperfect elasticity of the material. We note that here, just as in [1], in the inertial terms \( U_i, U_3, U_{ij}, U_{ij,2}, U_{ij,3} \) energy dissipation is not taken into account.

From (4)-(9), neglecting account for the deformations of dynamic contraction \( (\chi_3 = 0) \), Poissonian contraction \( (\chi_2 = 0) \), and also transverse shear \( (\chi_1 = 0) \), we can obtain differing lamellar shell theory variants, including the variant corresponding to the classical Kirchhoff–Love hypotheses.

In the particular case of an isotropic lamellar plate \( (\delta_{ij}^{(3)} = \delta_{ij}^{(2)} = \delta_{ij}^{(1)} = \delta_{ij}) \), neglecting the contraction deformations and the terms with the principal curvatures \( K_i \), but with account for the deformations of transverse shear in each layer, from (4)-(8) we have:

\[
\begin{align*}
N_{ij} - U_i - \varepsilon F_1 &= 0; \\
M_{ij} - U_{ij,1} - U_3 - \varepsilon F_2 - \varepsilon \cos pt &= 0; \\
M_{ij}^{(3)} - Q_{ij}^{(3)} - U_{ij}^{(3)} - \varepsilon F_3 &= 0,
\end{align*}
\]  
where \( \varepsilon \) is a small parameter, \( p \) is the frequency of the forcing load;

\[
F_1 = \pm \frac{3}{8}(1 + 2\cos \theta - \cos^2 \theta) \cdot \int_0^1 (\sigma_{ij,1}^{(3)} + \sigma_{ij,2}^{(3)} + \sigma_{ij,3}^{(3)} + \sigma_{ij,1}^{(2)} + \sigma_{ij,2}^{(2)}) \, \delta x^2 \, dx;
\]  

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