CALCULATING THE PRESSURE OF DUCTILE FAILURE OF A PIPE WITH AN AXIAL THROUGH CRACK

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A model is proposed of the threshold ductile state of a pipe with an axial crack with loading with internal pressure. The threshold plastic region is determined in the form of a rectangle with previously unknown boundaries. The strength conditions, linked with the maximum bending moment, is fulfilled at the boundary of the threshold region and the solution is continued to parts of the pipe away from the crack. The resultant theoretical values for the threshold pressure of ductile failure differ from the experimentally determined values by no more than 5-10%. The effect of axial load on the threshold fracture in pressure is analyzed.

The theory of the threshold state and the theory of brittle cracks represent a basis of modern fracture mechanics. These are mathematically completed theories which have been used as a basis for solving many problems of considerable importance for practice. These theories enable idealized description of the properties of ductile and brittle failure (plasticity and brittleness) which are typical to various degrees of all solids [1, p. 23].

Until recently these theories were developed in parallel, each for a specific grade of material without specifying the boundaries of the applicability. For example, it was proposed to determine the fracture pressure for a surface axial crack in a pipe simultaneously using two criteria [2]. However, the state of the material (brittle, ductile) is not a fixed fact and may vary in relation to temperature, loading rate, crack length, type of loading etc. [3], and transition from the brittle to the ductile state (and vice versa) does not take place discretely but gradually with transition to an intermediate quasibrittle region.

The currently used two-criterion approach is [4] in fracture mechanics described the threshold state of a body with a crack regardless of the dependence on the nature of loading. An advantage of the method developed in [5, 6] is that using the theory of the threshold state and the theory of brittle cracks as the boundary theories, clear limits are introduced to the applicability of each theory by indicating in the individual components the basis of the two-criterion approach of the diagrams of evaluating failure (DEF) of the brittleness Tbr and ductility Td point. The start of this division was proposed in [7] where the “brittleness number” \( \nu^2 \) was introduced:

\[
\nu^2 = \left( \frac{K_{Ic}}{\sigma_T} \right)^2 \frac{1}{a},
\]

where \( K_{Ic} \) is the critical stress intensity factor; \( \sigma_T \) is the yield limit; \( a \) is the crack length.

As indicated in [7], \( \nu^2 \) is the main but not exhausting parameters determining the nature and criterion of failure. It is necessary to take into account the functional multiplier \( \Gamma \left( \frac{a}{W} \right) \) (\( a \) is crack length; \( W \) is the characteristic dimension of the body) which depends no longer on the characteristic of the material but of the calculated values of \( K_I \) and \( P_{b_n} \), i.e., on a specific consideration of the bodies [8, 9].

For successful application of the two-criterion approach it is essential to solve an entire set of problems:

a) more accurate construction of DEF, especially of its transition regions;

b) calculation of the actual values of \( K_I \);

c) determination of the limiting load of ductile failure \( P_{b_n} \).

While the first two problems has been studied extensively in literature, insufficient attention has been paid to calculating \( P^b_n \) in fracture mechanics. For example, in a recently published four-volume monograph edited by V. V. Panasyuk [10] and entire volume is devoted to the results of determining \( K_I \) whereas no section has been devoted to \( P^b_n \) although as early as in 1968 Drucker noted that "the majority of failures of steel ... structures which resembled and are described as brittle failures, fail in fact at the threshold load or under plastic yielding condition" [11].

At the same time, the last edition of the standard R6 [4] stresses that it is important to calculate \( P^b_n \) as a compulsory component in the two-criteria approach. Increasing interest in calculating \( P^b_n \) is indicated by an excellent review studied by A. G. Miller [12]. However, the solutions presented in this review do not reflect the entire variety of real geometries and they are only of empirical nature for important cases in practice. For example, for a pipe with an axial continuous crack the literature gives mainly empirical equations based on statistical data [2, 4]. The first comment indicating that at task to calculating \( P^b_n \) for this geometry has already been specified was made as early as in 1968 [11], but only several statistically possible solutions have been published since [13, 14] and they give too low results [15] and do not take into account the effect of axial load.

Therefore, it is important to obtain new solutions for the threshold load of ductile failure of solids with cracks. This should result not only in more accurate calculation of ductile structures but should also make the two-criteria approach in strength calculations more popular. In this work we propose a model of the threshold state of a pipe with an axial continuous crack which is then used to calculate the pressure of ductile failure \( P^b_n \).

The model described below is a further development of a model of plastic deformation of a pipe with an axial surface crack. Assuming that the tangential stresses are equal to zero as a result of restricted displacement of the points of the pipe caused by the presence of a binder (nonzero net section), a statically possible solution was obtained previously [16].

For a thin-walled pipe with an axial external right-angled surface crack the implicit expression for \( P^b_n \) has the form [16]

\[
\frac{2\lambda^2 d_l}{(\alpha - \lambda_i)} \left(1 - \frac{t_h}{t} \right) = 1 - (1 - 2\beta)^2 + (1 - \beta - \frac{t_h}{t})^2 H \left(1 - \beta - \frac{t_h}{t} \right) ;
\]

\[
\lambda^2 = \frac{P^b}{R^2} ;
\]

where \( l \) is the crack length; \( R \) is the radius; \( t \) is pipe thickness, \( t_h = t - d \), \( d \) is the defect depth; \( \alpha \) is the coefficient of reduction of strength caused by the presence of a crack,

\[
\alpha = \frac{P^b R}{\sigma_{ul}} ;
\]

\( \beta \) is the intensity of axial load \( N_x \),

\[
\beta = \frac{N_x}{\sigma_{ul}} ;
\]

for a pipe closed at the ends \( \beta = \alpha /2 \); \( H(x) \) is Heaviside's function

\[
H(x) = \begin{cases} 0 & x < 0; \\ 1 & x \geq 0 . \end{cases}
\]

Assuming in Eq. (2) that \( t_h = 0 \), we obtain a statically possible solution for a pipe with closed ends and with a continuous defect:

\[
\frac{2\lambda^2 \alpha}{1 - \alpha} = 1 - (1 - \alpha)^2 + \left(1 - \frac{\alpha}{2} \right)^2 .
\]

The values of \( \alpha = \alpha(\lambda^2) \) is determined from the equation (2') give too low values in comparison with the experimental data, especially at \( \lambda^2 \geq 1 \). In fact, if \( \alpha \), determined from Eq. (2') corresponds to the actual fracture stress, then any local plastic failure, associated with surface defects would lead immediately to global failure. However, this is not always the case. The point is that divergence of the pipe walls is accompanied by the formation of tangential stresses.