RELATIONS DETERMINING CONTACT FORCES. PART 2.
WHAT IS AN ABSOLUTELY SOFT BODY?

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Examples are considered of attachment between bodies and the breakage of internal links, which illustrate connectivity transformation. If the displacements and the forces are combined and separated in the combination and separation of bodies, the connectivity transformation acquires the physical meaning of a rigidity transformation. Along with other definitions, one is given for the statics of an absolutely compliant (absolutely soft) body.

Halves of Points. In each body, we consider two types of points: internal and boundary ones, which differ primarily in the topological sense. On account of this, they differ in freedom of motion and are subject to different forces.

To give this scheme a clear form, we represent the internal points by integers and the boundary ones by halves, as in Fig. 1a.

Consider as an example the space of real numbers $\mathbb{R}$. In that case, the continuity axiom states that if the set of all real numbers is divided into two nonempty sets $X$ and $Y$ not having common elements and such that for any points $x \in X$, $y \in Y$ one has $x < y$, then there exists a unique number $\xi$ (boundary) for which $x \leq \xi \leq y$ for any $x \in X$, $y \in Y$. This concept of integer and half-integer points provides the physical content of this axion. We assert that if one sections the number axis at some point, then a certain point necessarily falls at the point of section, which is divided into two parts. Usually, the point of section relates to set $X$ or to set $Y$ and one obtains one set open and the other set closed. We on the other hand assign one half of the point to set $X$ and the other to set $Y$. Consequently, the two sets formed on the section are closed, as are the bodies in accordance with the definition.

We now consider two bodies whose boundaries consist of half-integer points and which are in contact. The half points of one body closely adjoin the half points of the other, so they thus form as it were integer points. The set of all these integer points constitutes the contact set.

When we combine the bodies, we cement together the halves of the points that are in contact. This gives integral internal points. They can always be distinguished from the other internal points, which were integral ones and are obtained not by cementing. The internally linked points are cemented, and can under certain circumstances be broken into halves.

The arithmetic of the internal and boundary points becomes clearer when the two halves constitute an integer part and vice versa. We perform the operations of combining and separating the bodies and count all the halves. These may be separated or cemented in some way into pairs. However, the total number is always conserved and invariant with respect to the connectivity transformations.

Consider two halves in space $E_3$. Let them lie in such a way that one of them occupies position $\bar{x}$ and the other $\bar{y}$. The displacement of the first is the vector $\bar{u}$ and that of the second is $\bar{v}$ (Fig. 1b). The halves come into contact when

$$\bar{x} + \bar{u} = \bar{y} + \bar{v}$$

(1)

(the displacements coincide)
Condition (1) is necessary but not sufficient for the two halves to form an integral point. It is required that the halves form an equilibrated point in order that it should be an integral one.

Let one half be in a state of equilibrium under the external forces $F_1$ and $-F_1$ while the second is the same under forces $F_2$ and $-F_2$ (Fig. 1c). When one combines the halves, two of these four forces are transformed into an internal interaction and the other two represent the external load for the integral point. One should have

$$f_1 = -f_2$$

Then only when conditions (1) and (2) are obeyed can one form an integral point.

Half points represent a conditional name. If the points for example are considered in a plane and are integral circles, then the boundary points are not halves but quarters and so on. When the bodies are combined, internal forces arise between the internal parts and between the pieces of boundary. The internal-connection points in the interiors in the plane in that case consist of halves, while the internal connection points on the pieces of boundary will be formed as the union of two quarters (Fig. 2).

**Examples of Body Adhesion and Bond Breakage.** There are many cases where one considers body adhesion or bond breakage. These are cases of collision and contact, when the bodies adhere or, conversely, when cracks propagate or slip or detachment surfaces are formed, or when there is loss of adhesion between parts of a body on failure. This includes situations where for example a deformable body is subject simultaneously to the addition and removal of material at various parts of the surface or form the same part but at different times.

Two cases are considered below. The first is an example of adhesion between bodies, namely compression of two contacting bodies, which is a solution to the Hertz problem, while the second differs from the first in that one considers bond breakage, which is a simple example from failure mechanics.

**Example 1.** Compression of two contacting bodies (adhesion in the Hertz problem). The geometrical aspect on the contact between two bodies in accordance with the Hertz treatment has been considered previously [1, 2]. Here we present the corresponding static treatment [3].

We assume that the bodies at the point of contact have spherical surfaces with radii $R_1$ and $R_2$ (Fig. 3a). If there is no pressure between the bodies, there is contact at one point $O$. The distance can be measured from a plane tangential at $O$ to points $M_1$ and $M_2$ at a small distance $r$ from the $z_1$ and $z_2$ axes and represented with sufficient accuracy by

$$z_1 = \frac{r^2}{2R_1}; \quad z_2 = \frac{r^2}{2R_2}.$$  

(3)

If the bodies are compressed along the $z_1$ and $z_2$ axes by a force $P$, local strains arise at the point of contact, which lead to contact over a certain small surface having a circular boundary. If points $M_1$ and $M_2$ come into contact as a result of local compression, then

$$z_1 + z_2 = \frac{r^2(R_1 + R_2)}{2R_1R_2}.$$